Il Caos Dinamico,
ovvero
l'Aleatorieta' Deterministica
e
la descrizione dinamica dei fenomeni

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Determinismo: definizione

Causal determinism is, roughly speaking, the idea that every event is necessitated by antecedent events and conditions together with the laws of nature.

Determinism is deeply connected with our understanding of the physical sciences and their explanatory ambitions, on the one hand, and with our views about human free action on the other.

In both of these general areas there is no agreement over whether determinism is true (or even whether it can be known true or false), and what the import for human agency would be in either case.
Hoefer, Carl, "Causal Determinism",
The Stanford Encyclopedia of Philosophy

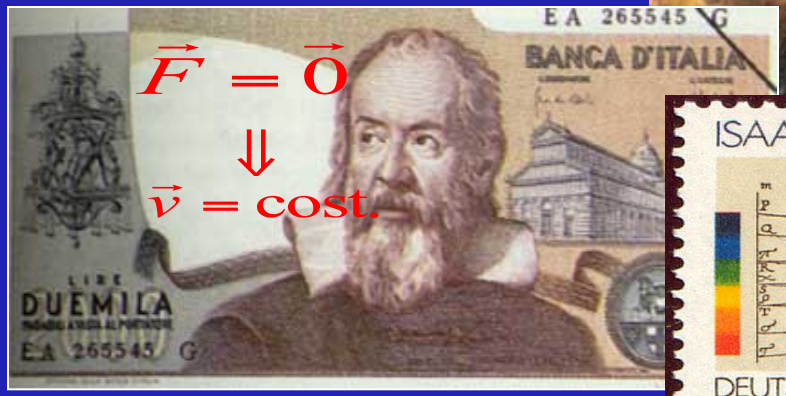
Le Leggi della dinamica

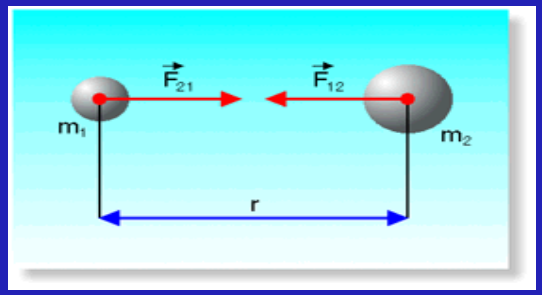
δύναμις = forza (greco antico)

Le forze producono il moto

$\vec{v}$ $\vec{F}$

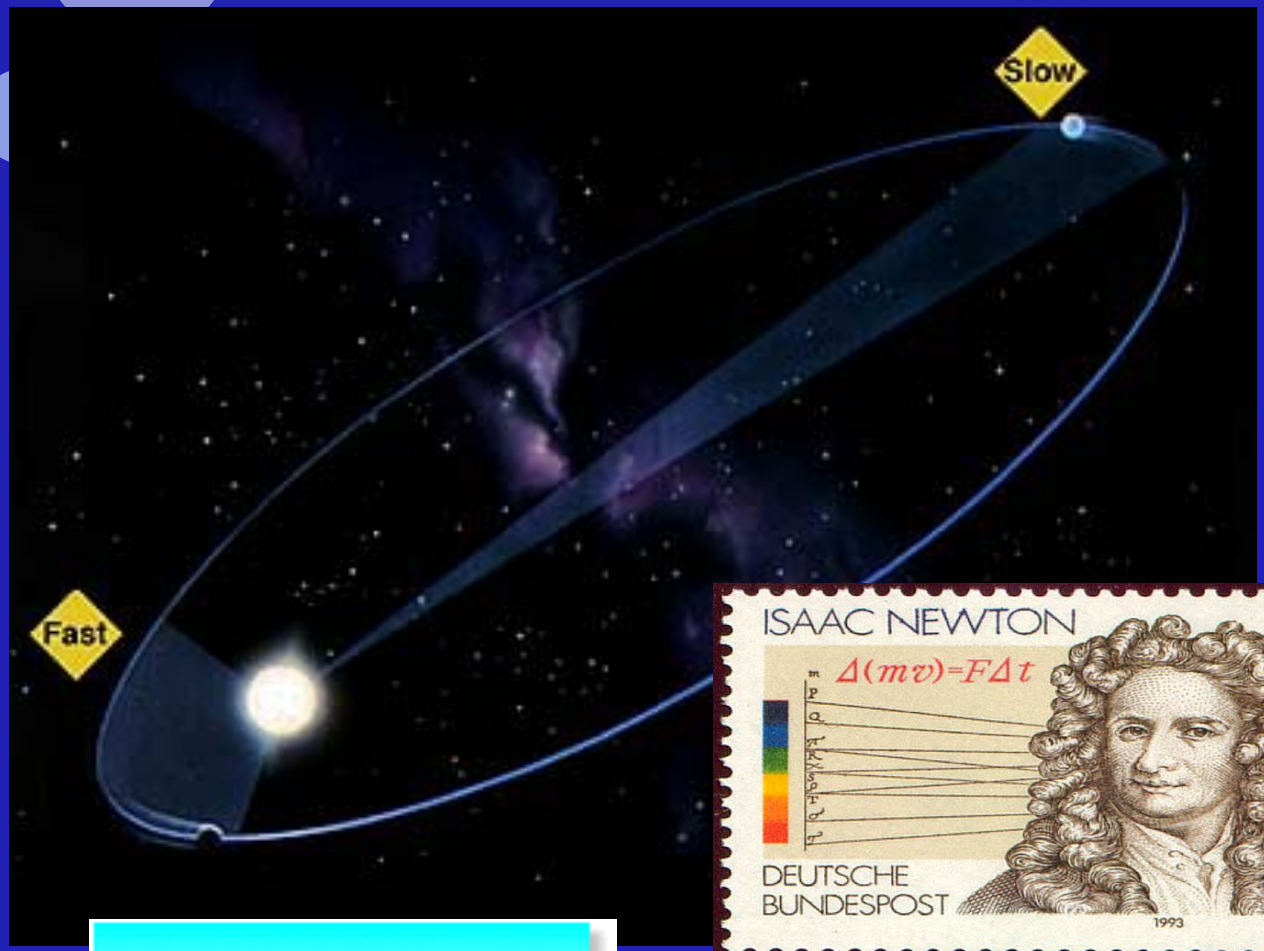
$$\vec{v} \propto \vec{F}$$



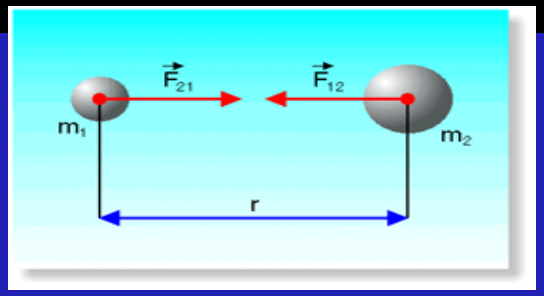


$$\vec{F} = m_1 m_2 \frac{\vec{r}}{r^3}$$

La meccanica celeste

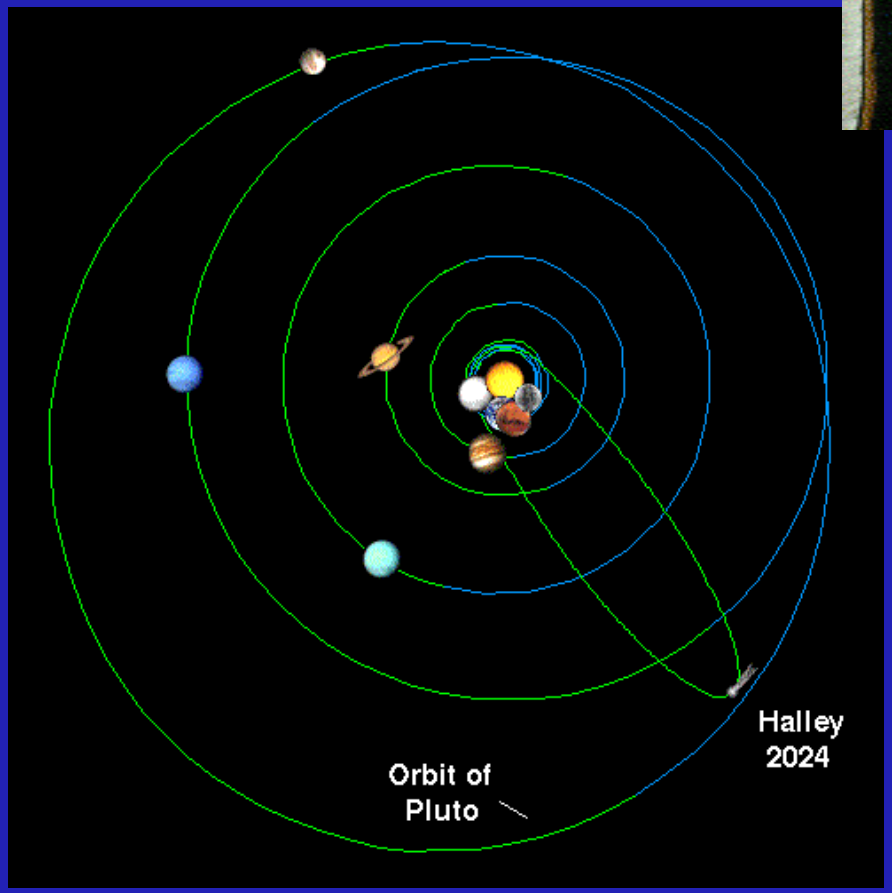


$$\vec{F} = m\vec{a}$$



$$\vec{F} = m_1 m_2 \frac{\vec{r}}{r^3}$$

Le comete



La lunga coda della cometa del 1456 sparse il terrore nell'Europa. La conoscenza delle leggi del sistema del mondo, acquisita in questo intervallo di tempo, ha dissipato i timori prodotti dall'ignoranza dei veri rapporti dell'uomo con l'universo; ed Halley, riconosciuta l'identità della cometa con quella del 1531, 1607 e 1682, annunziò il suo ritorno per la fine del 1758 o l'inizio del 1759.

Pierre Simon de Laplace

Determinismo laplaciano

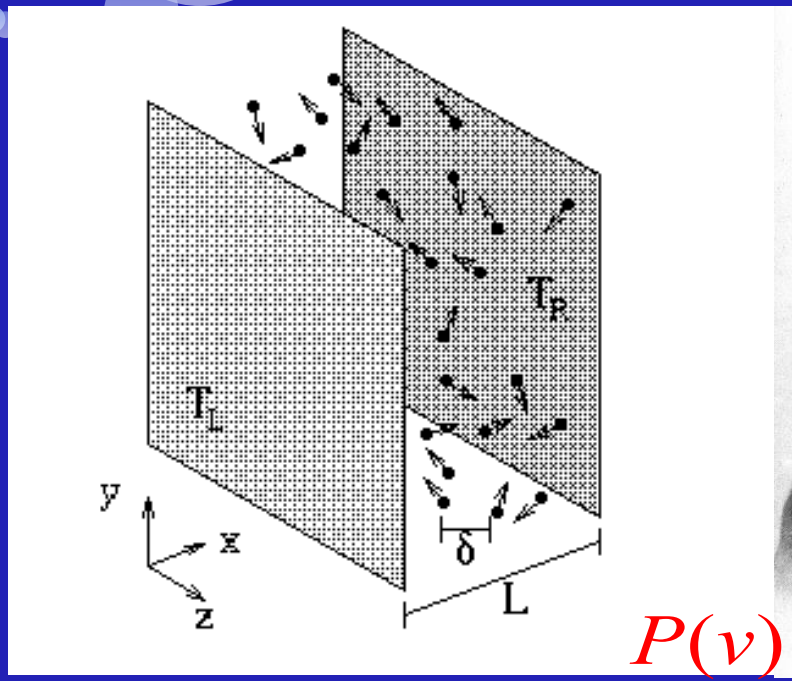
Lo stato attuale della Natura e' evidentemente una conseguenza di quello che essa assumeva nell'istante precedente.

Se concepiamo una intelligenza che, ad un dato istante, possieda tutte le relazioni delle entita' dell'Universo, essa potrebbe determinare le posizioni relative, le velocita' e le relazioni di causalita' di tutte queste entita' ad ogni istante del tempo, passato o futuro.

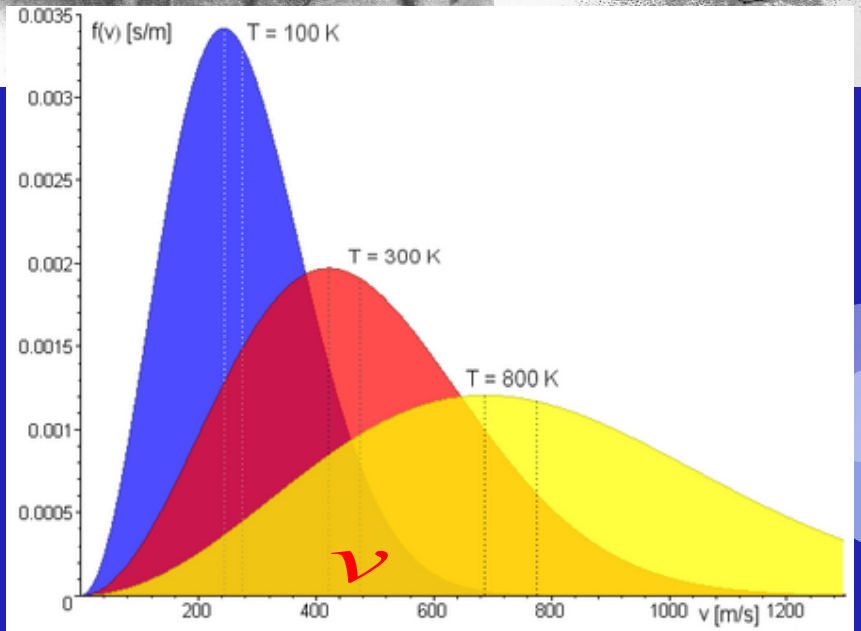
L'Astronomia, il ramo della scienza che piu' rende onore alla mente umana, ci offre una idea, per quanto imperfetta, di cosa una tale intelligenza dovrebbe essere.

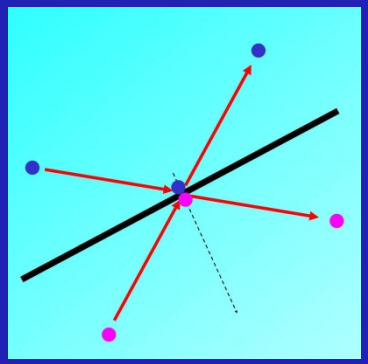
P.S. Laplace, 1814

La teoria cinetica dei gas



$P(v)$





Stosszahlansatz

Maxwell

E' una dottrina metafisica che dagli stessi antecendenti seguano le stesse conseguenze. E non e' di molta utilita' in un mondo nel quale gli stessi antecedenti non capitano mai, e niente accade due volte.

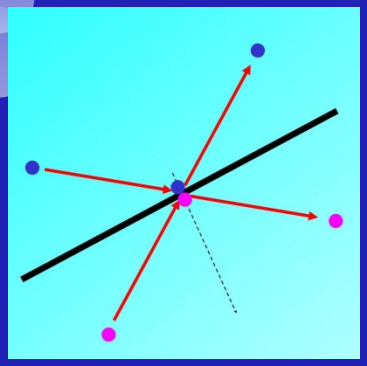
Il principio fisico che assomiglia a questo e' che da antecedenti simili seguano conseguenze simili. Siamo passati da uguaglianza a similarita', da accuratezza assoluta ad una approssimazione piu' o meno precisa.

Ci sono classi di fenomeni ove un piccolo errore nei dati introduce un piccolo errore nei risultati: il corso degli eventi in questi casi e' stabile. Ci sono altre classi di fenomeni ove si verificano fenomeni di instabilita'.

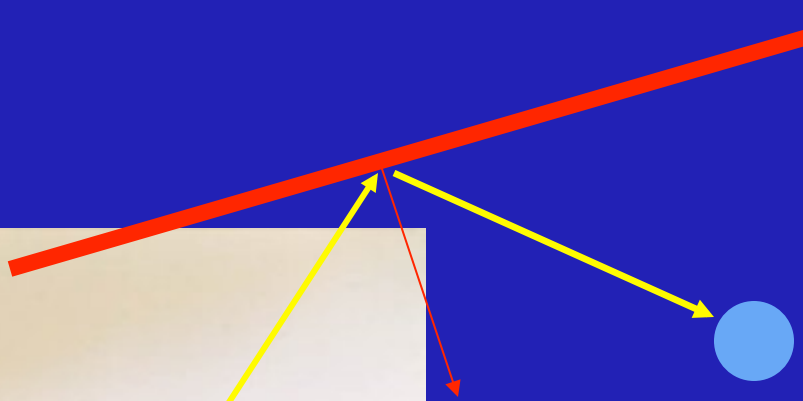
La vera logica di questo mondo e' il calcolo delle probabilita'.

J.C. Maxwell, 1860

La teoria del gioco



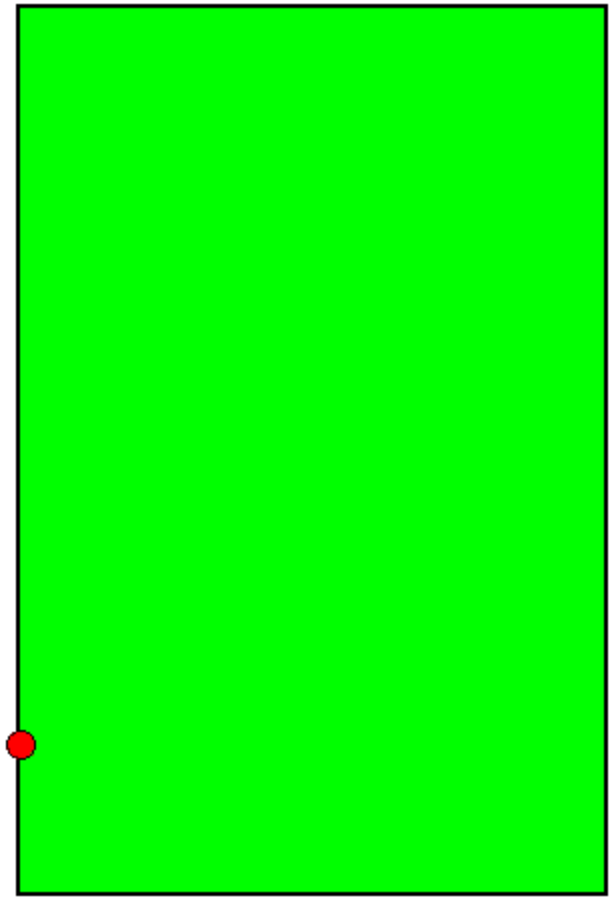
Legge della riflessione elastica



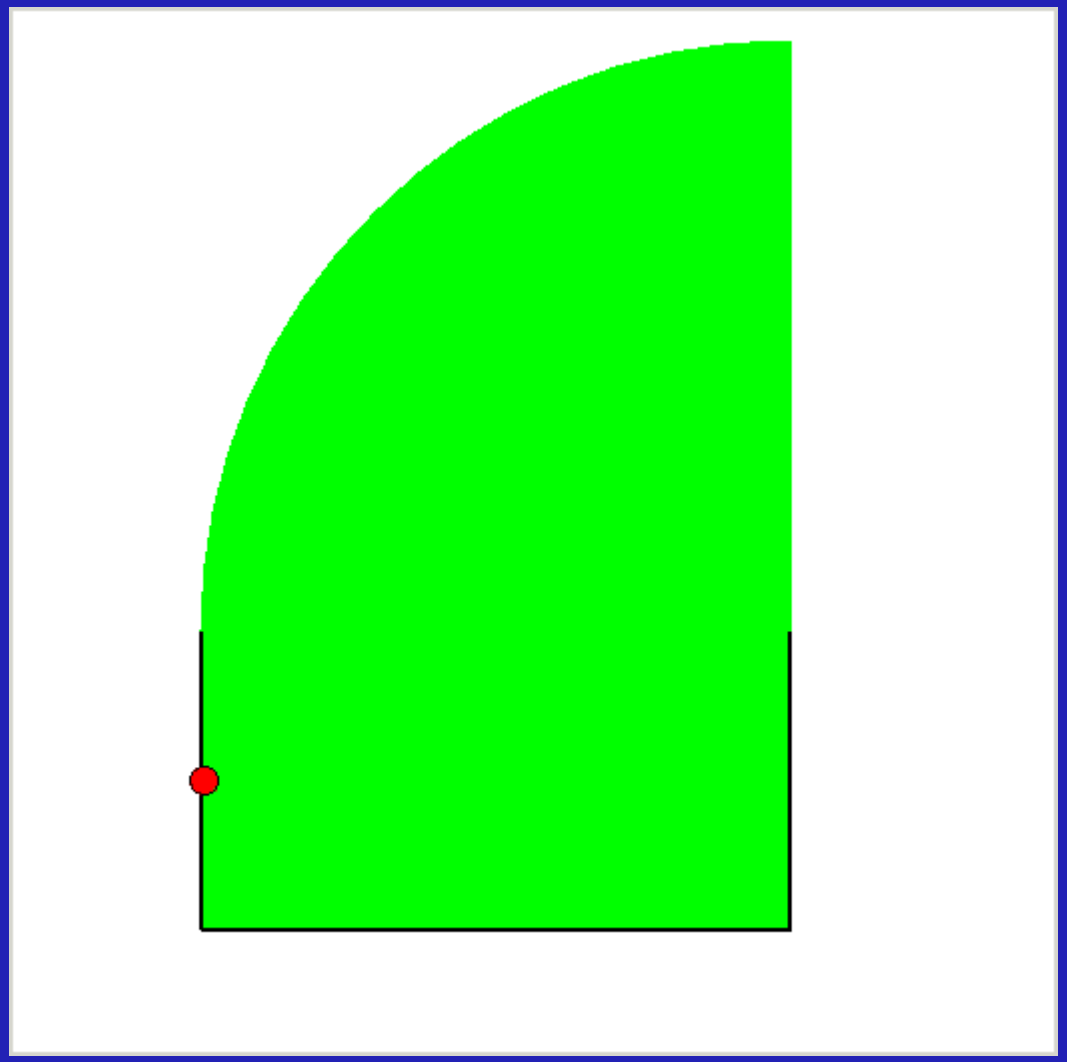
Stosszahlansatz

Sala Biliardo

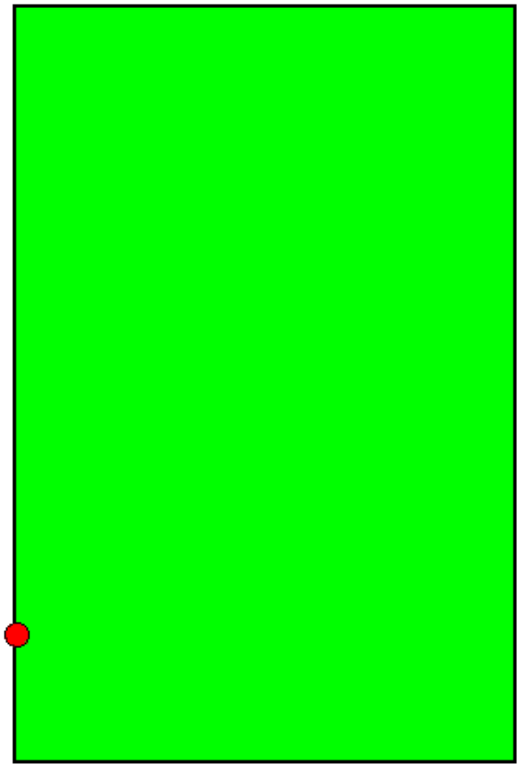
I biliardi rettangolari

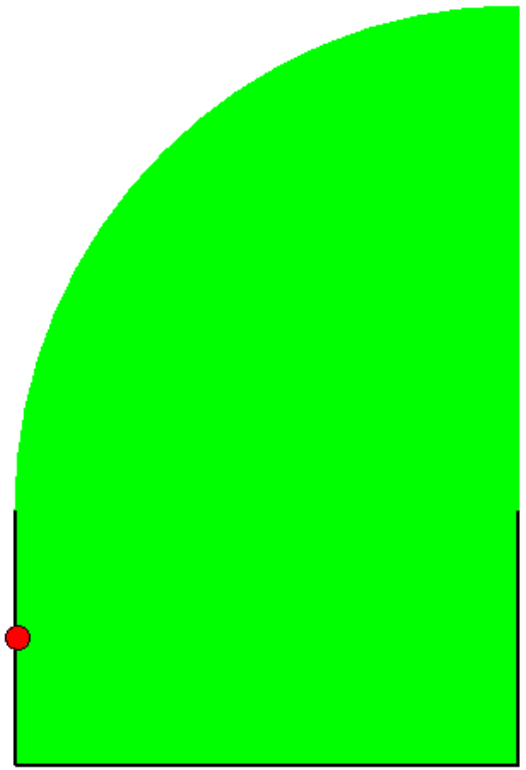


I biliardi curvilinei



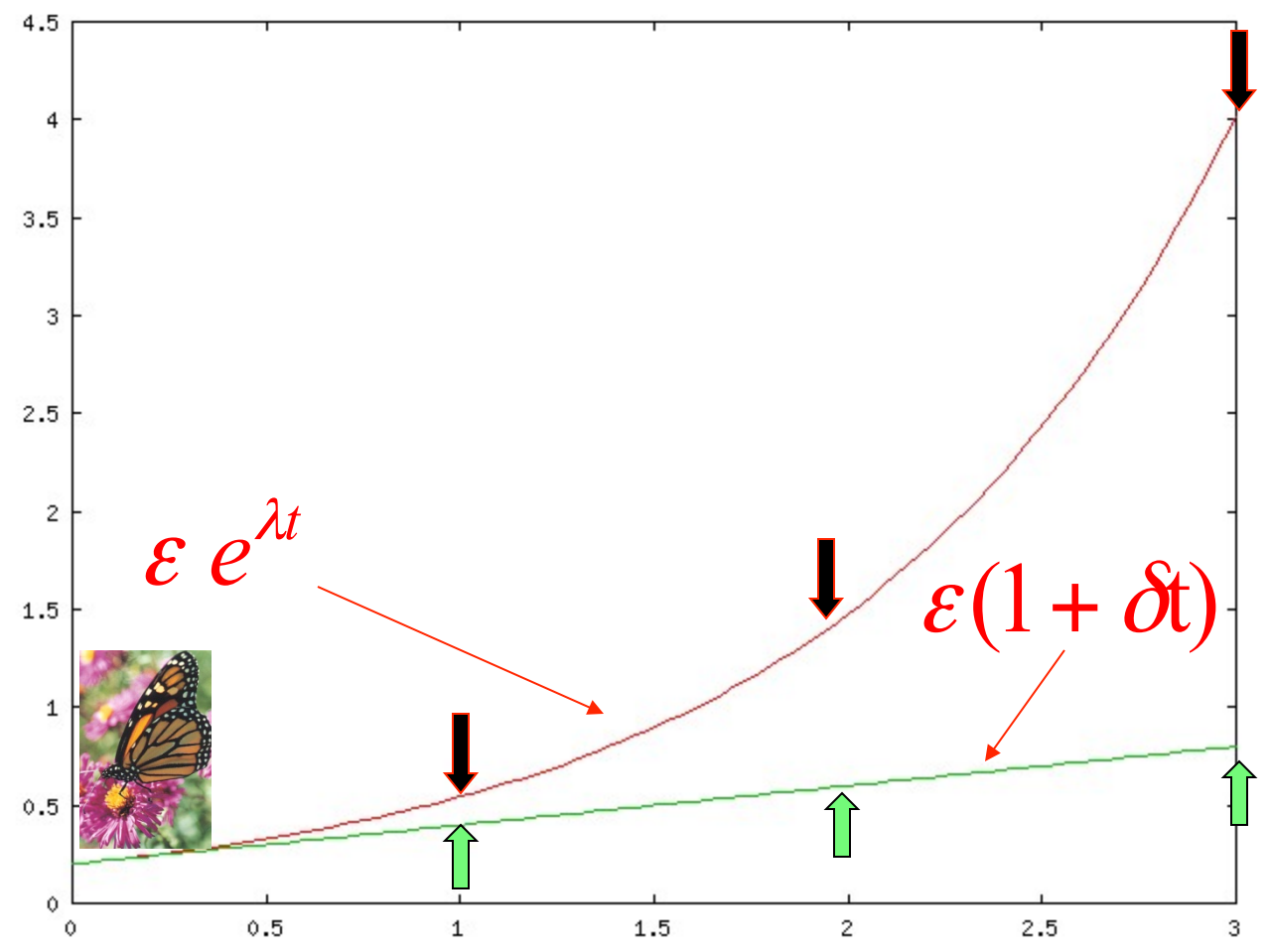
I biliardi





In quale aspetto quantitativo consiste la differenza tra i due moti?

Separazione lineare verso esponenziale



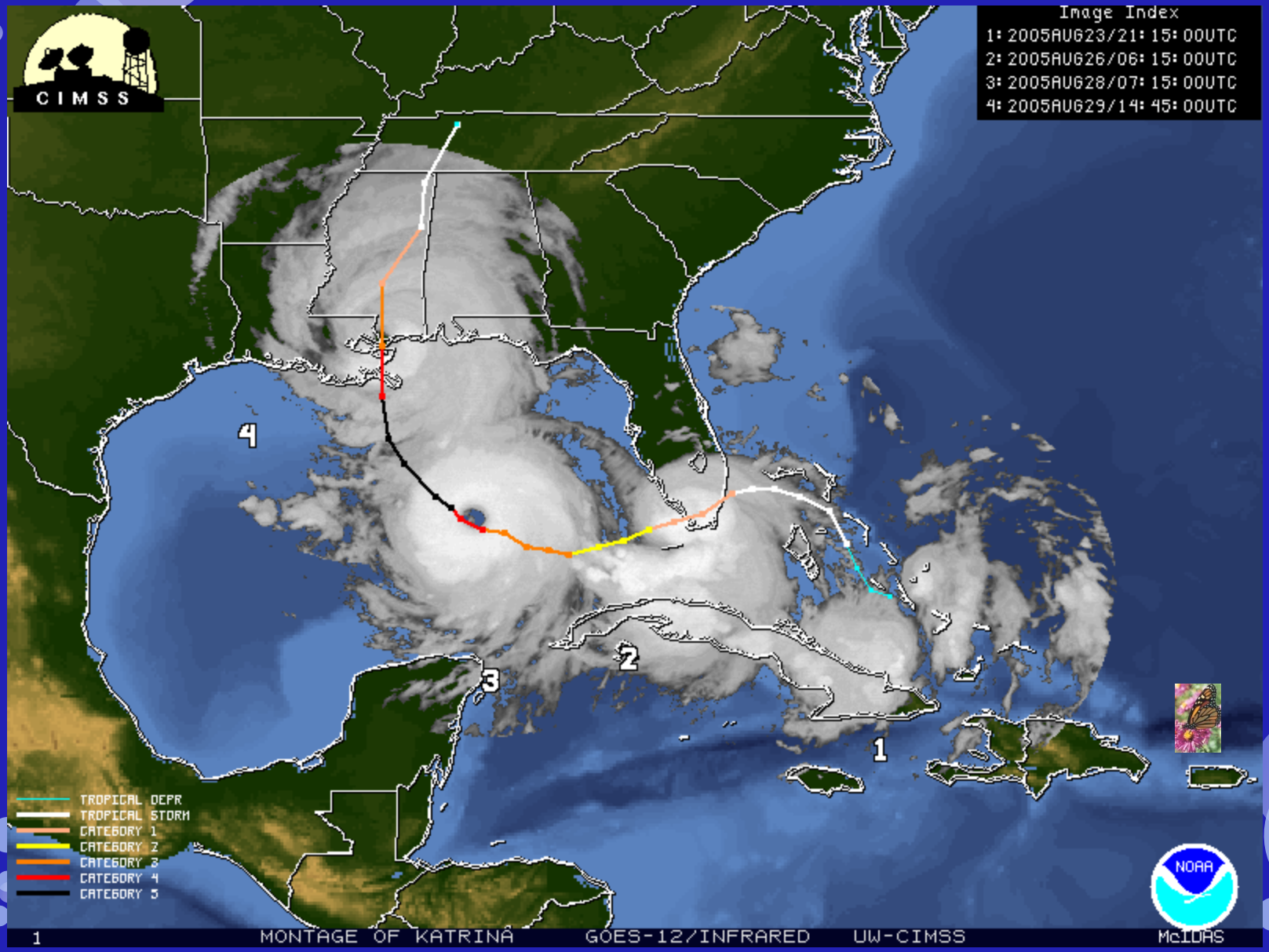
Butterfly effect

Hollywood !!

PARK™

World Year of PHYSICS 2005
www.physics2005.org

New Orleans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2: 2005AUG26/06:15:00UTC
3: 2005AUG28/07:15:00UTC
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4
3
2
1
TROPICAL DEPR
TROPICAL STORM
CATEGORY 1
CATEGORY 2
CATEGORY 3
CATEGORY 4
CATEGORY 5
NOAA
1
MONTAGE OF KATRINA
GOES-12/INFRARED
UW-CIMSS
McIDAS

Jules Henri Poincare'

Poincare'

Quando una piccola causa che sfugge alla nostra osservazione determina un effetto considerevole, noi diciamo che questo effetto e' dovuto al caso.

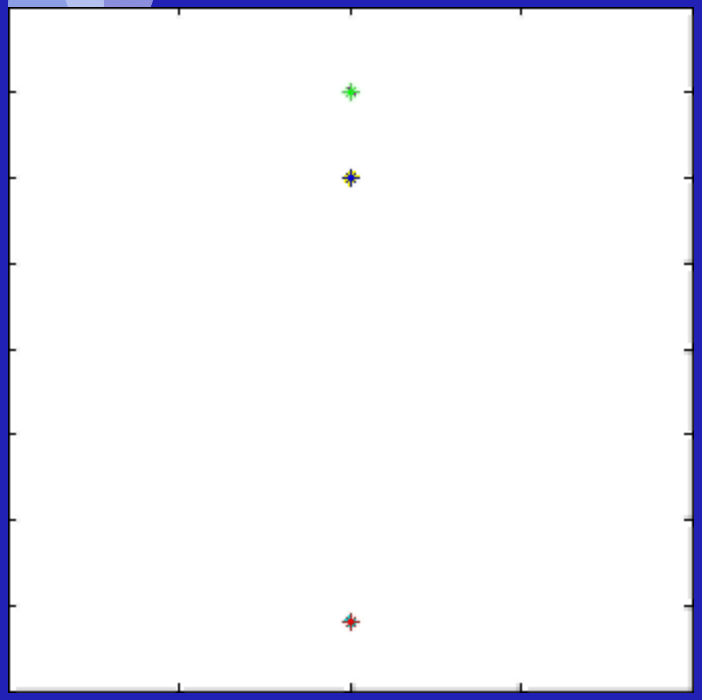
Anche se le leggi della natura non avessero per noi piu' alcun segreto, tuttavia noi potremmo conoscere la situazione iniziale solo in modo approssimato.

Se questo ci permettesse di predire la situazione successiva colla stessa approssimazione, potremmo dire che il fenomeno e' stato predetto. Puo' invece succedere che una piccola differenza nelle condizioni iniziali causi un enorme errore nelle seconde. La predizione diviene impossibile, e questo e' il fenomeno casuale. Il determinismo e' una fantasia dovuta a Laplace.

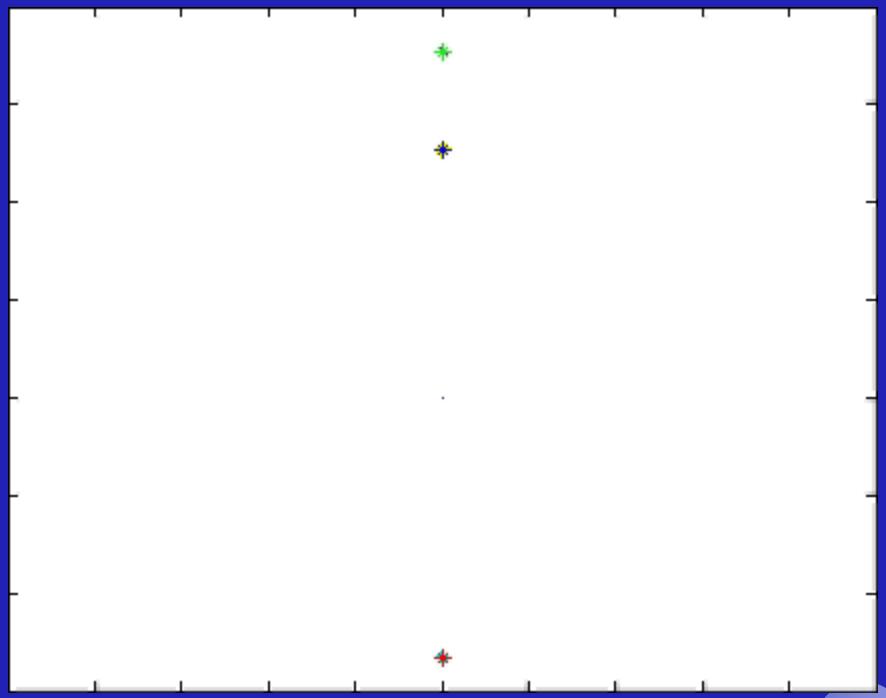
J.H.

Poincare', 1903

Caos nel sistema solare: il problema dei tre corpi

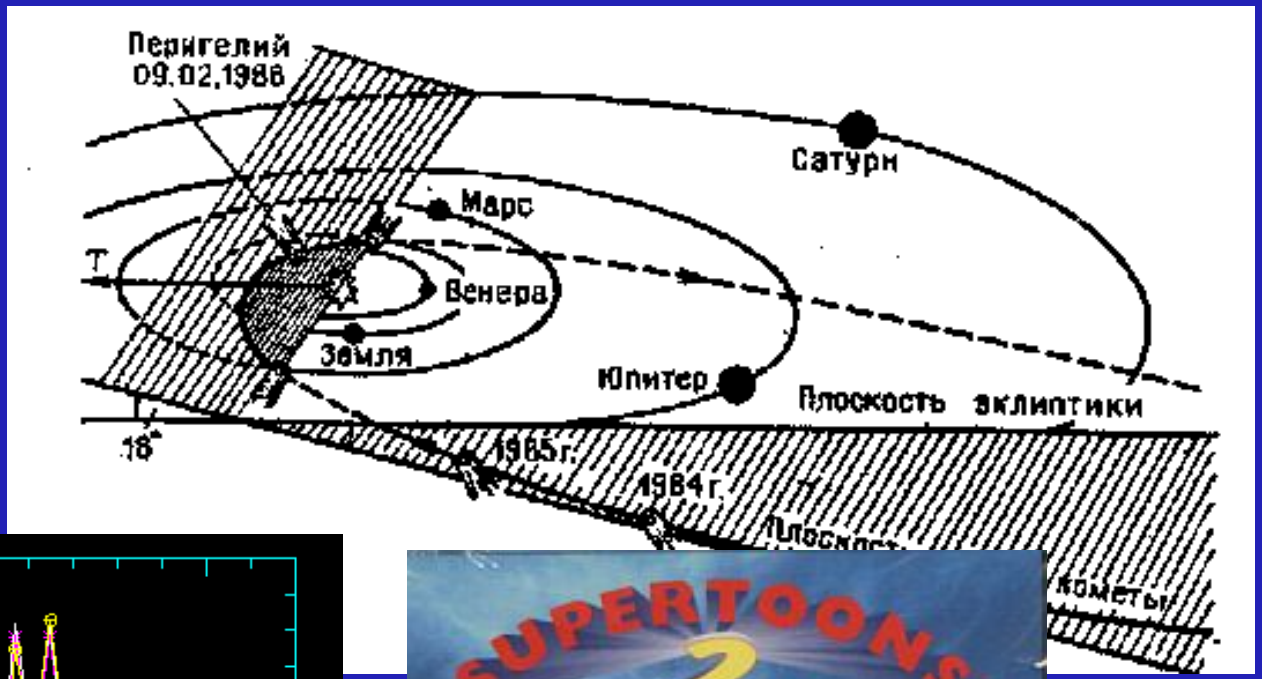


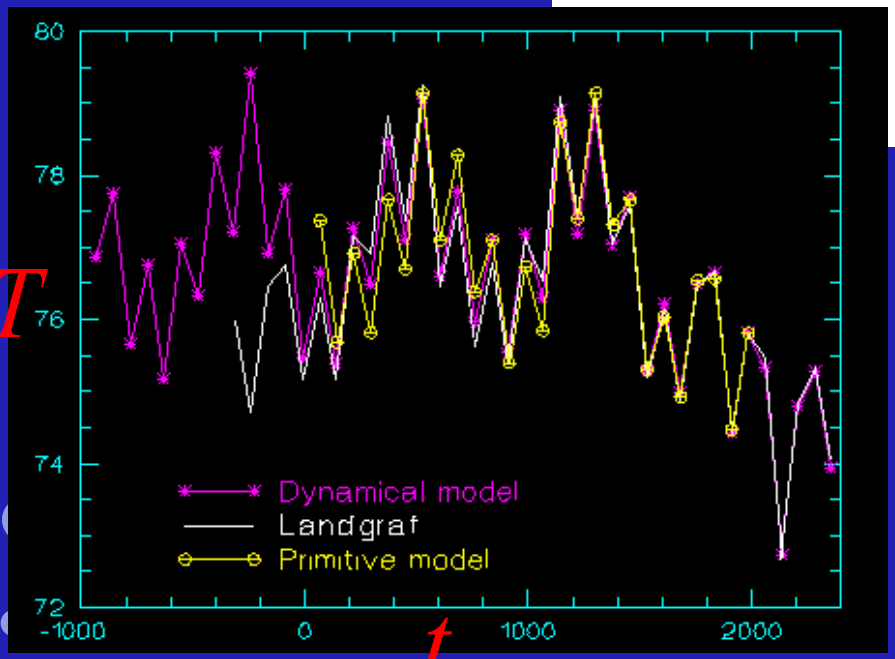
Sole = 333480 m,
Giove=317.89 m

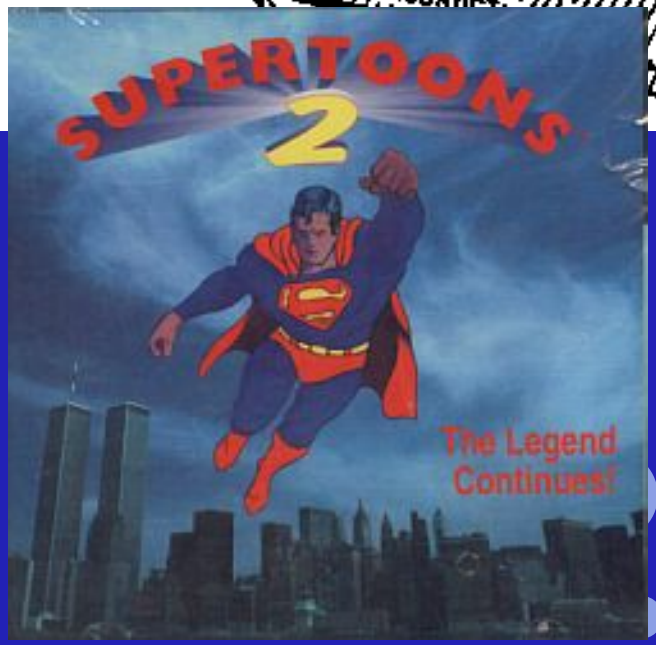


Sole = 333480 m,
Giove=317890 m

La cometa di Halley e' caotica

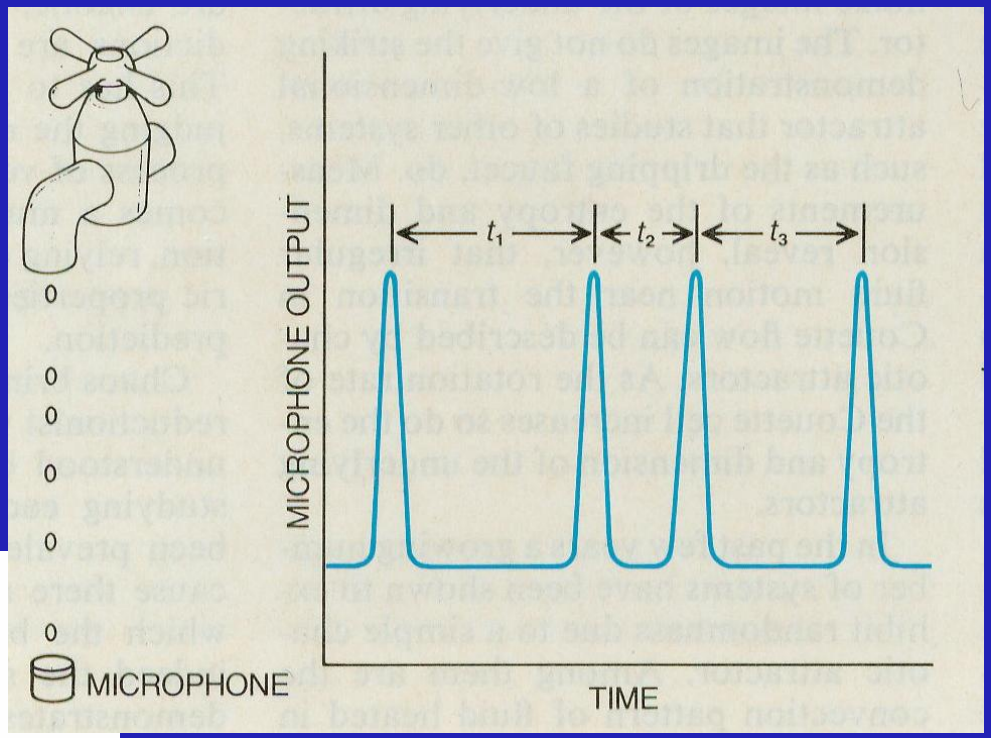


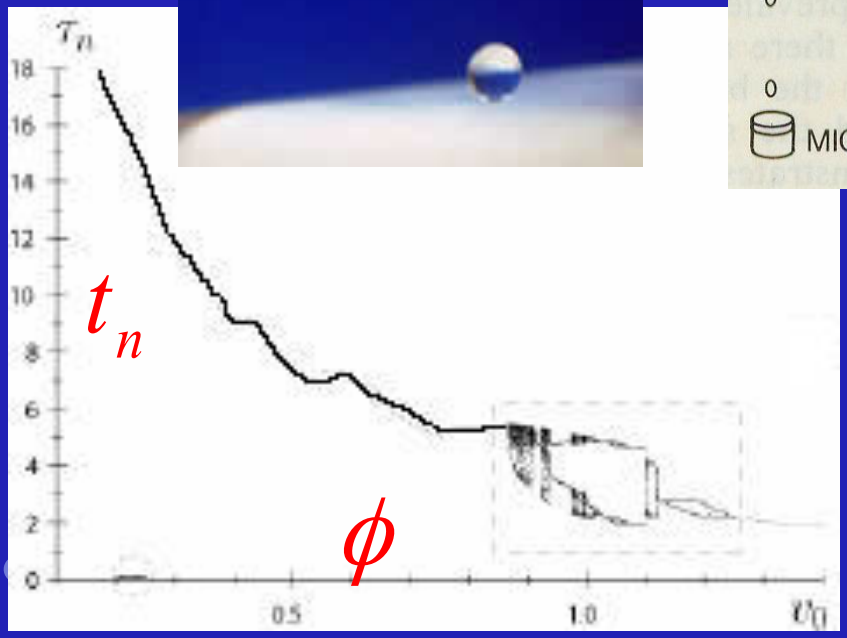




Il caos e' ovunque

In cucina…





Aleatorieta' Deterministica

Sequenze aleatorie

$$\sigma = \sigma_0\sigma_1\sigma_2\ldots\sigma_n\ldots$$

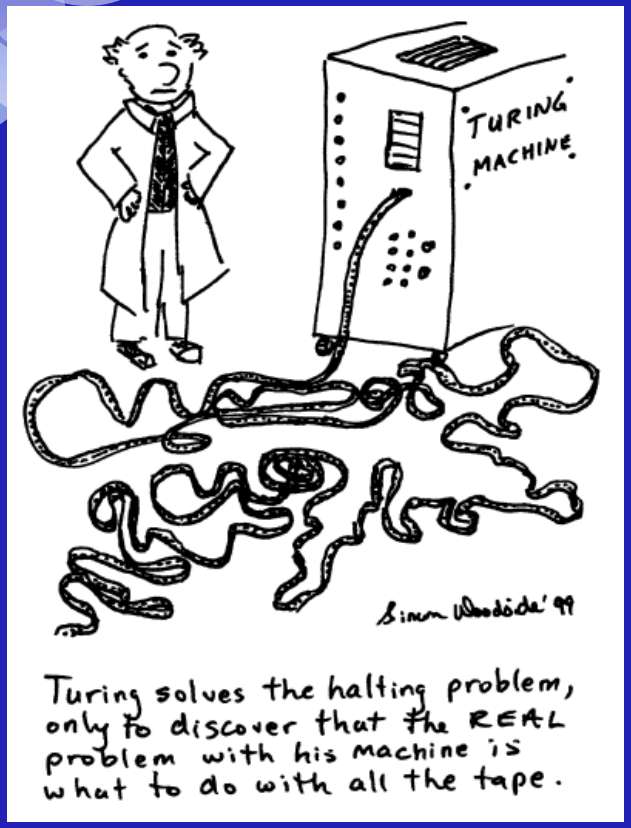
lettere di un alfabeto

→ frasi

$\sigma =$ 3.14159265358979323846264338327950..

$\sigma =$ 6.65800370588543407290934194390186..

Kolmogorov-Chaitin Complexity



Programma

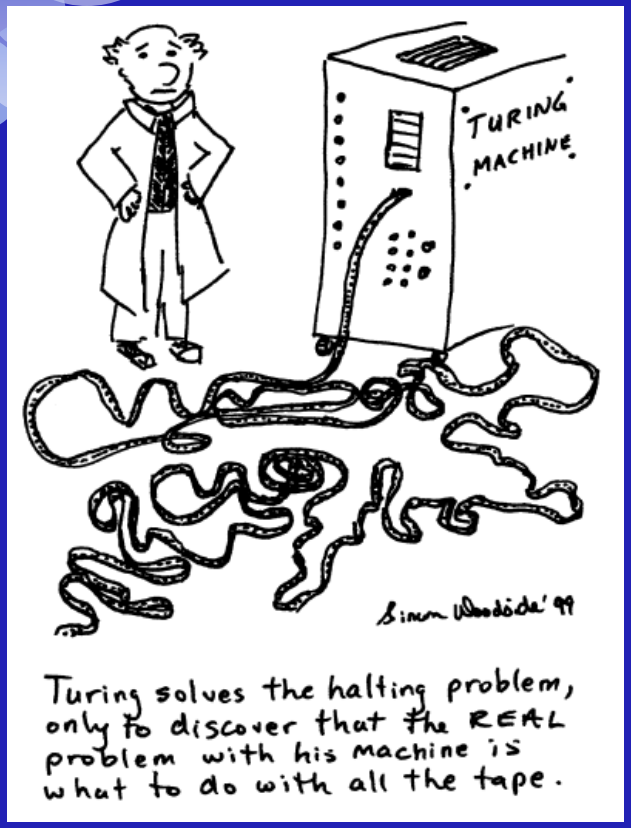
$$\theta = \theta_0\theta_1\theta_2\theta_3\ldots\theta_k$$

$$\sigma = \sigma_0\sigma_1\sigma_2\ldots\sigma_n\ldots$$

Risultato del calcolo

$$\sigma = insettochfailmieleronzavivendlalveare$$

Kolmogorov-Chaitin Complexity



Programma

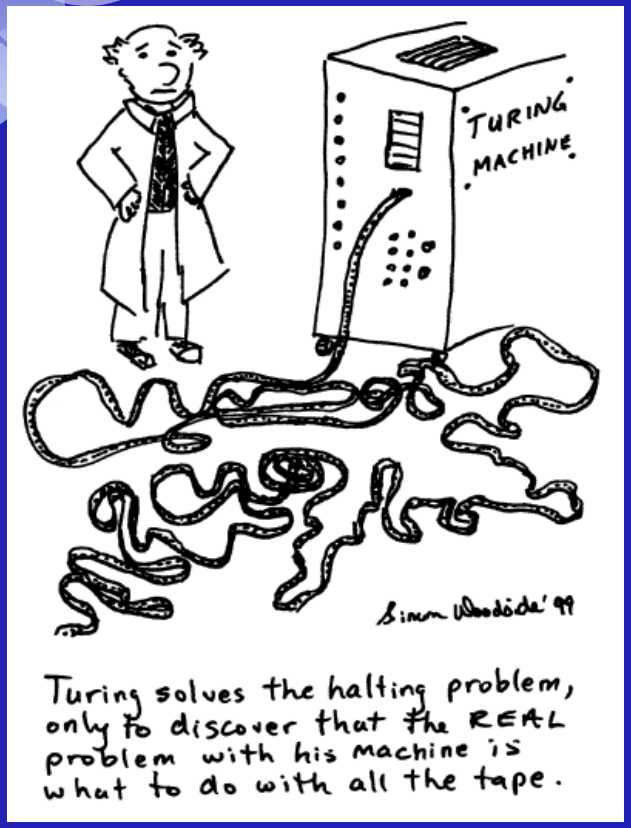
$$\theta = \theta_0\theta_1\theta_2\theta_3\ldots\theta_k$$

$$\sigma = \sigma_0\sigma_1\sigma_2\ldots\sigma_n\ldots$$

Risultato del calcolo

$$\sigma = insettochefailmieleeronzavivendalveare$$

Kolmogorov-Chaitin Complexity



Programma

$\theta =$ *perimetro/ 2 * raggio*

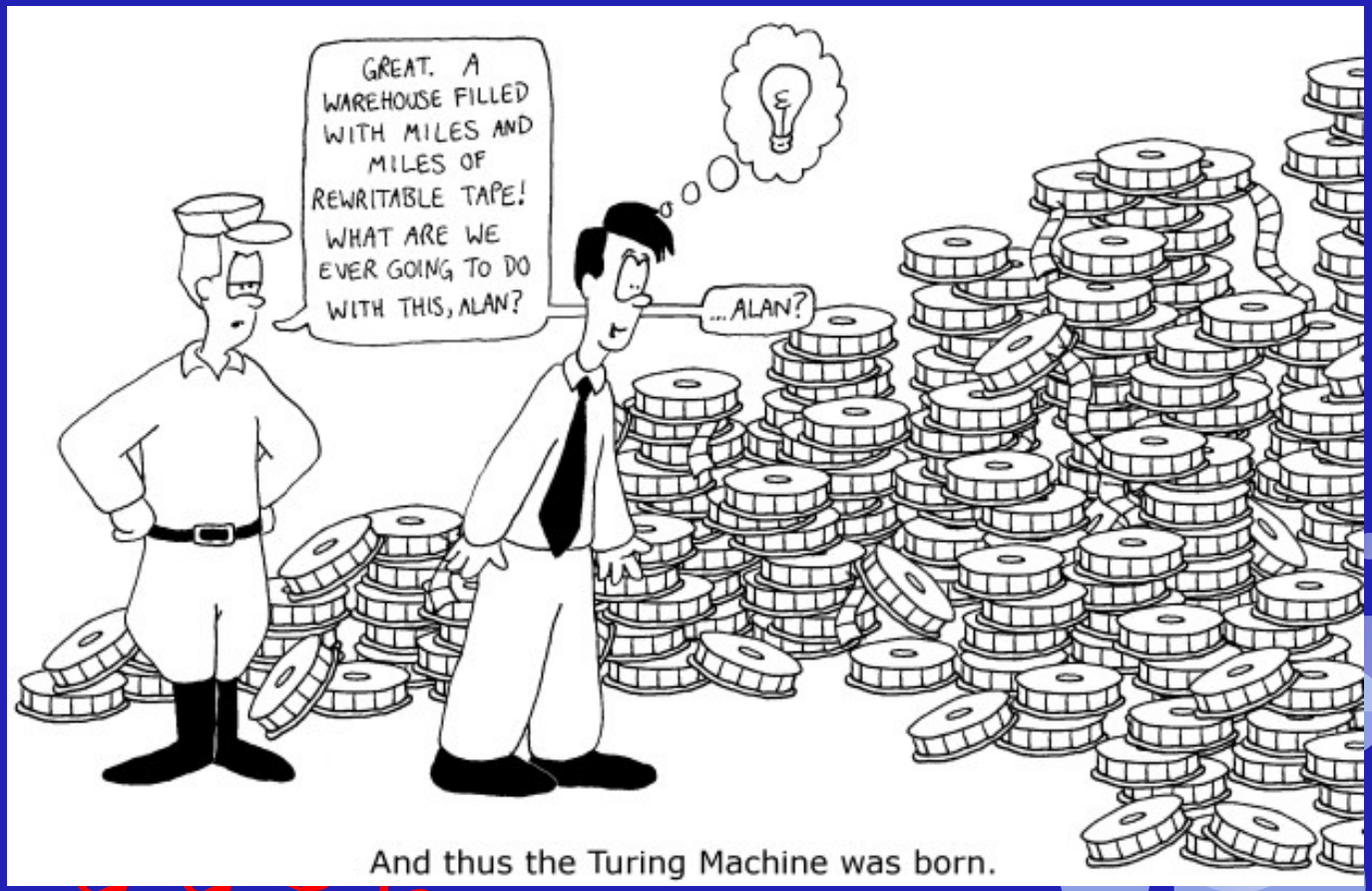
$\sigma =$ 3.14159265358979..

Risultato del calcolo: frase computabile

Turing halting problem

Tutti i programmi:

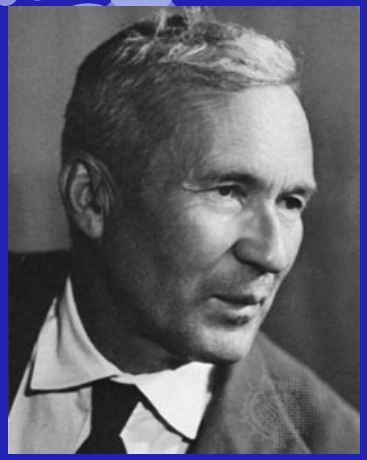
Tutti i risultati



Non computabile!

$$\xi = \xi_0 \xi_1 \xi_2 \xi_3 \xi_4 \cdots$$

Kolmogorov-Chaitin complexity



$$\eta = \eta_0 \eta_1 \ldots \eta_{(n)}$$

Piu' corto programma

$$\theta = \theta_0 \theta_1 \theta_2 \ldots \theta_{(k)}$$

K(n) = complessita' della frase

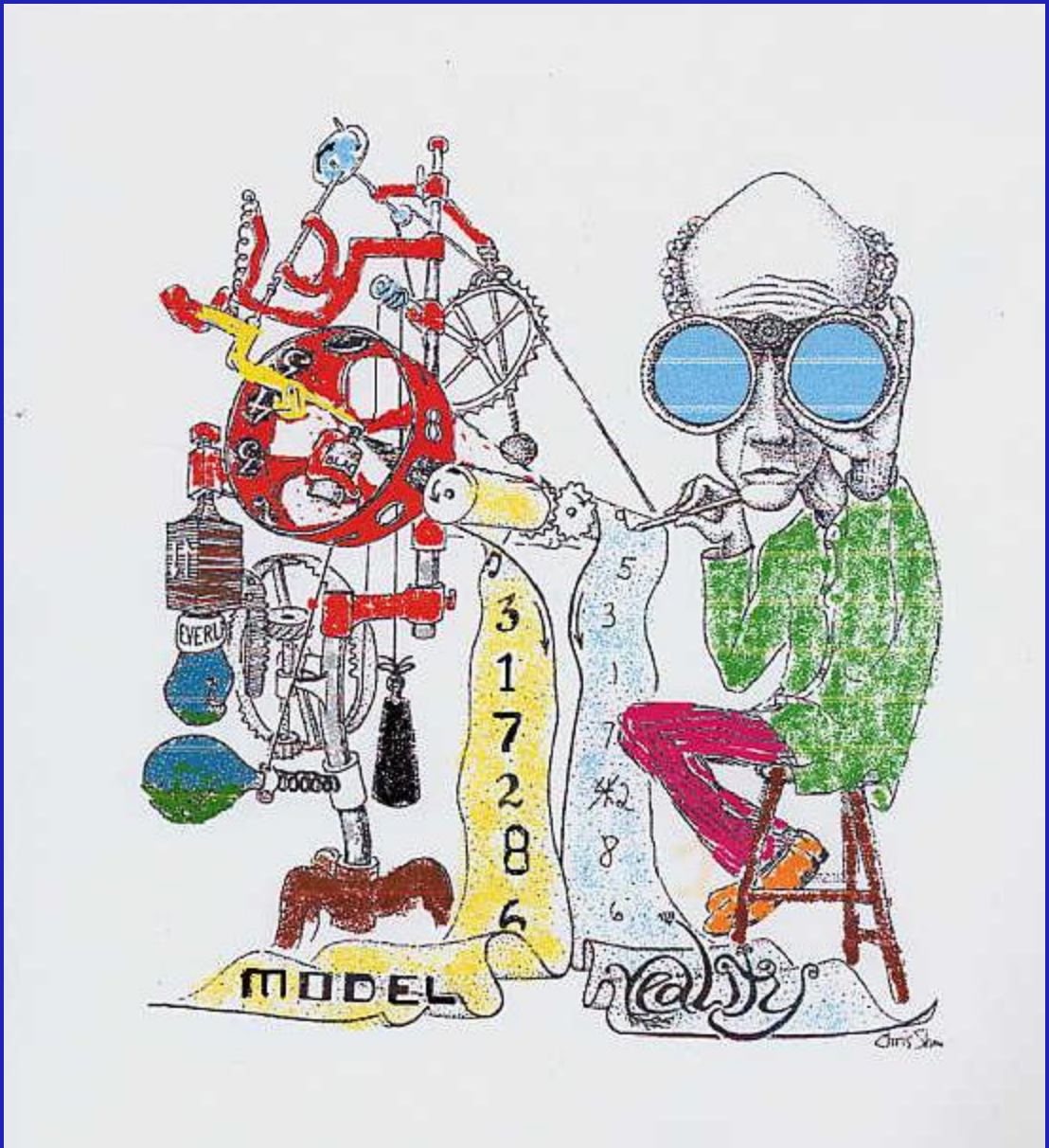
K(n) ~n frase non computabile, casuale

Sequenze aleatorie

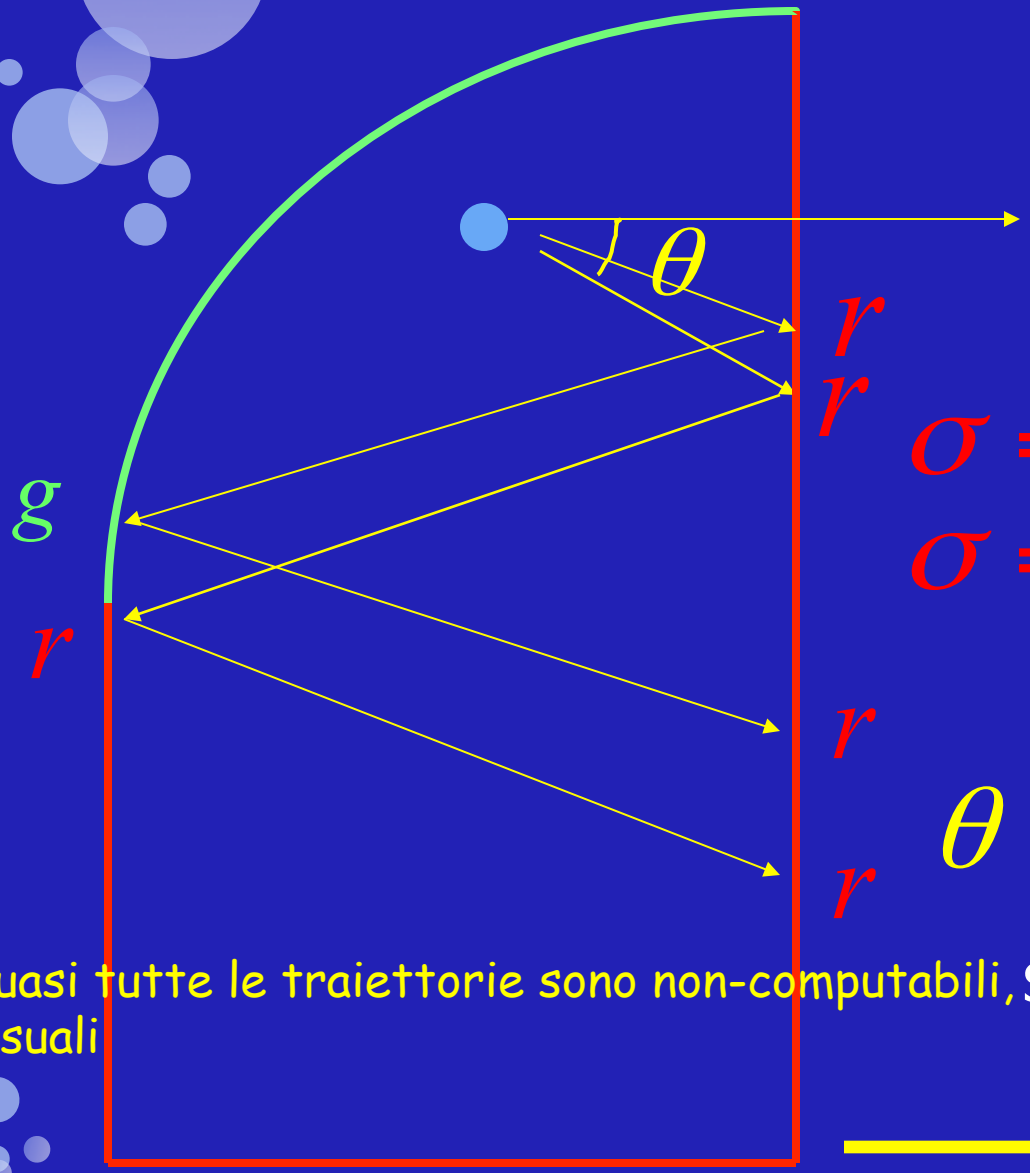
$$\sigma = \sigma_0\sigma_1\sigma_2\ldots\sigma_n\ldots$$

Quasi tutte le realizzazioni sono non-computabili, casuali

La dinamica simbolica



Dinamica simbolica nei biliardi



$$\sigma = rgr.... = 010....$$

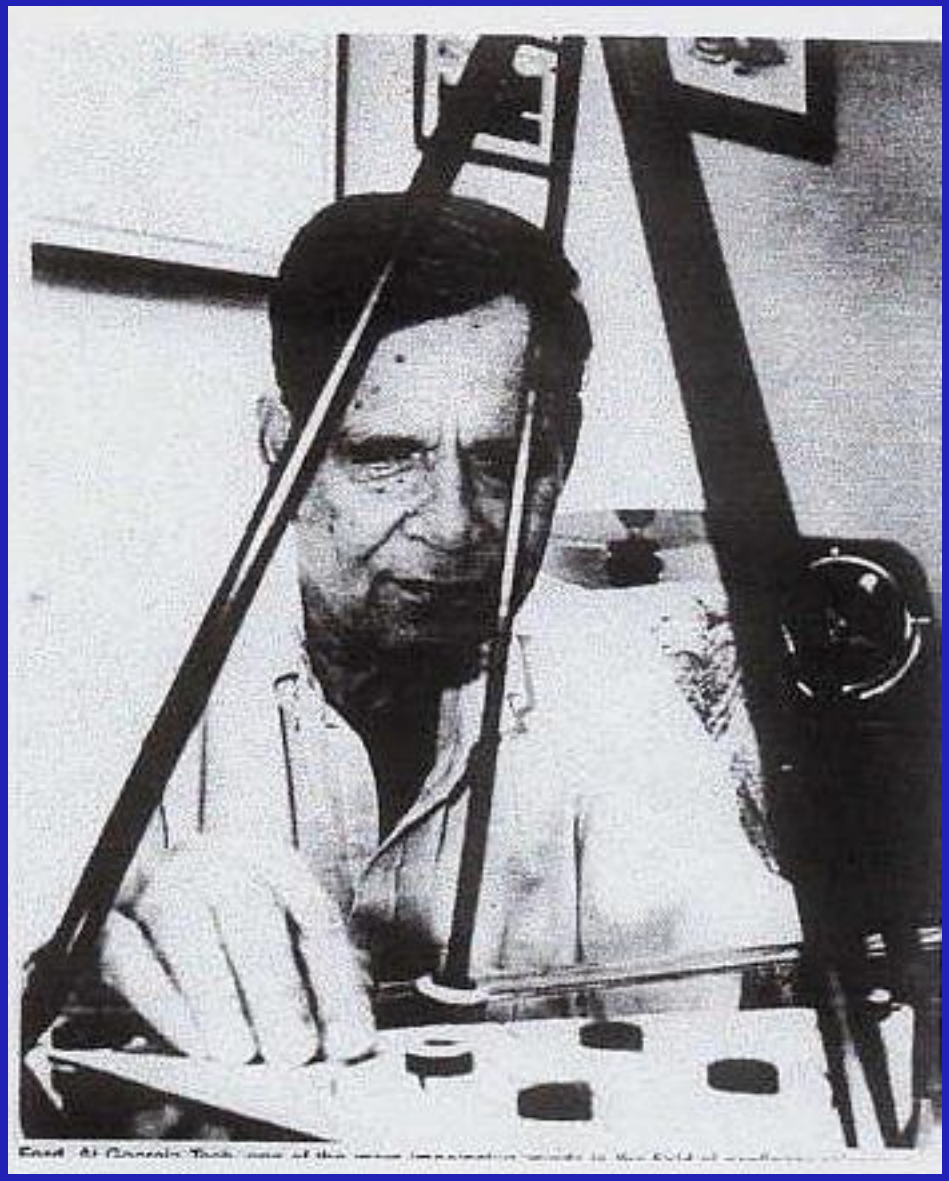
$$\sigma = \sigma_0\sigma_1\sigma_2...\sigma_n....$$

$$\theta = \theta_1.\theta_2\theta_3...\theta_k....$$

Quasi tutte le traiettorie sono non-computabili, Separazione esponenziale casuali

$$\longrightarrow \quad k \approx n$$

Joseph Ford



Ford

Il caos e' aleatorieta' deterministica.

Il caos e' la dinamica libera dal giogo dell'ordine e della predicibilita'. Esso permette ai sistemi di esplorare ogni loro possibilita' dinamica. E' varieta' eccitante, ricchezza di scelta, una cornucopia di opportunita'.

Il caos e' la terza rivoluzione scientifica di questo secolo.

J.Ford, 1986

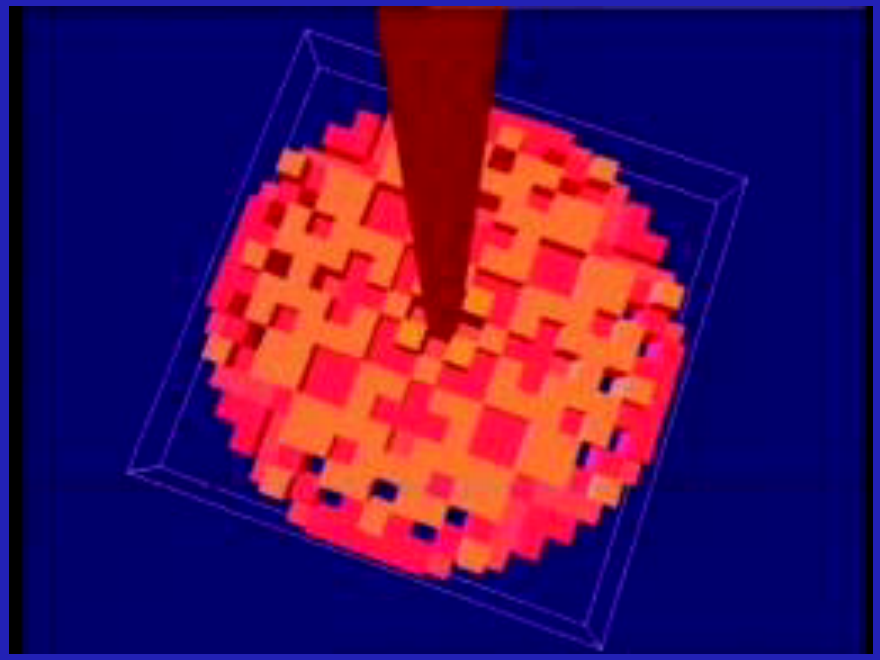
Riassunto

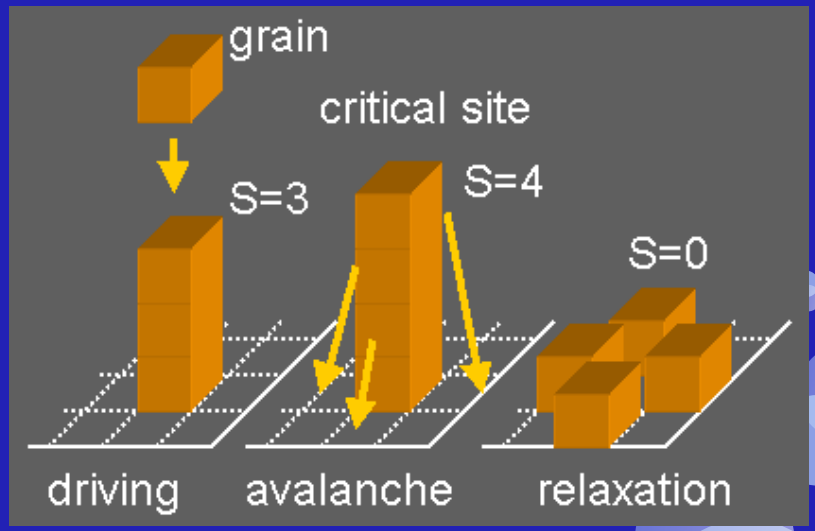
	Phase-space dimension	
dynamics	small	large
linear	integrable	integrable
Non-linear	Chaotic, occasionally integrable	Complex, seldom integrable, marginally unstable, long transients

Mucchi di sabbia, valanghe ..

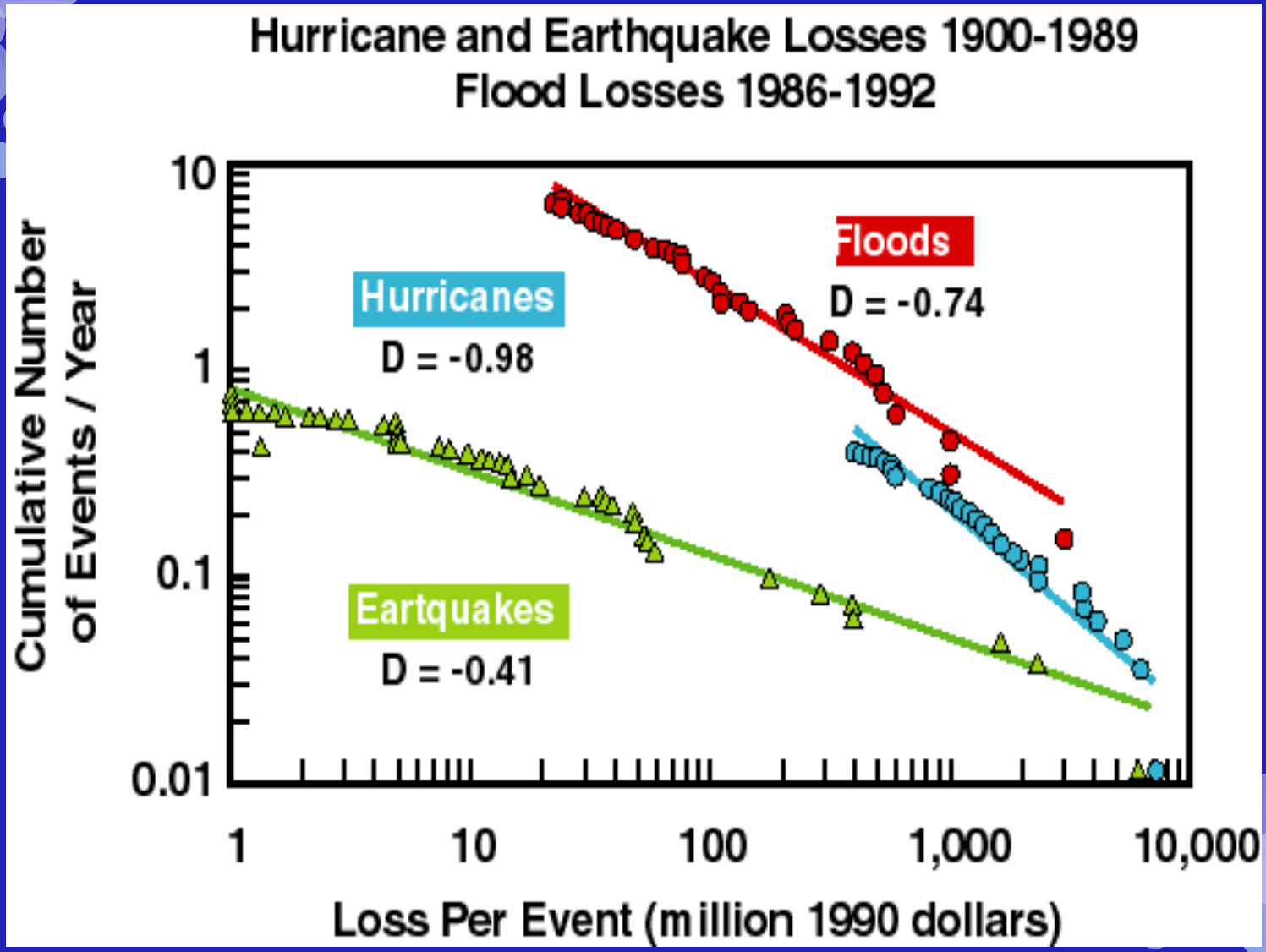
Sistemi a molte dimensioni, con regole semplici, non-lineari

Quando la pendenza e' superiore ad una soglia, I granelli si spargono

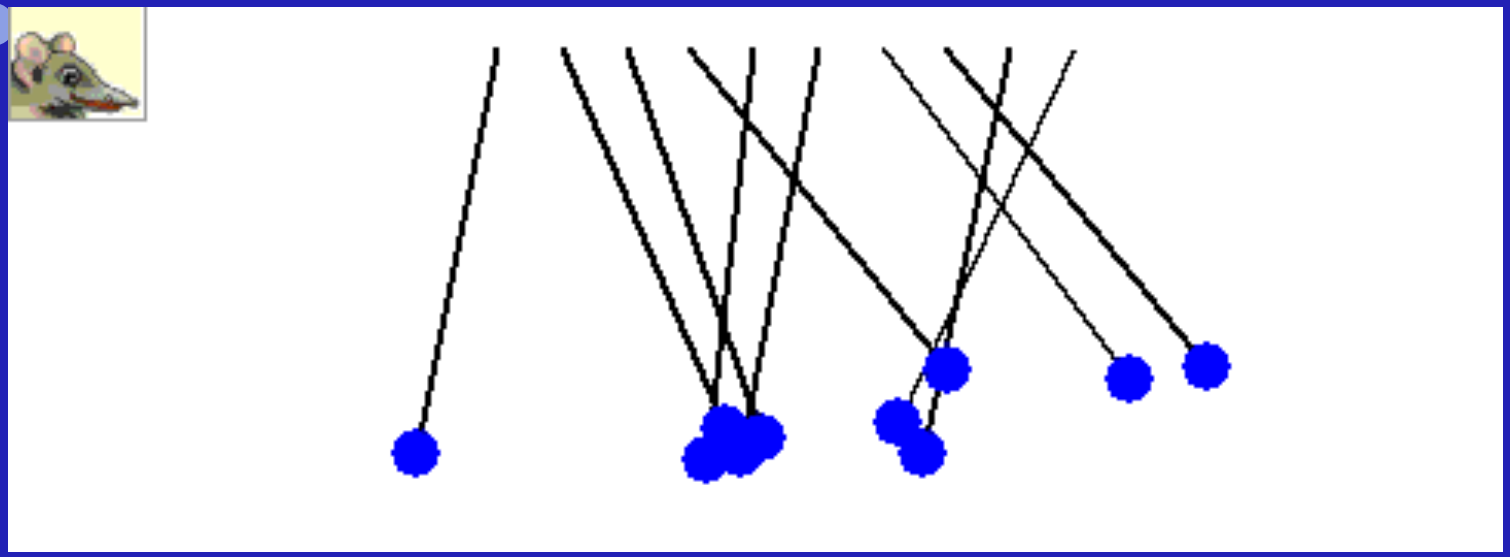




Calcoliamo la taglia delle valanghe

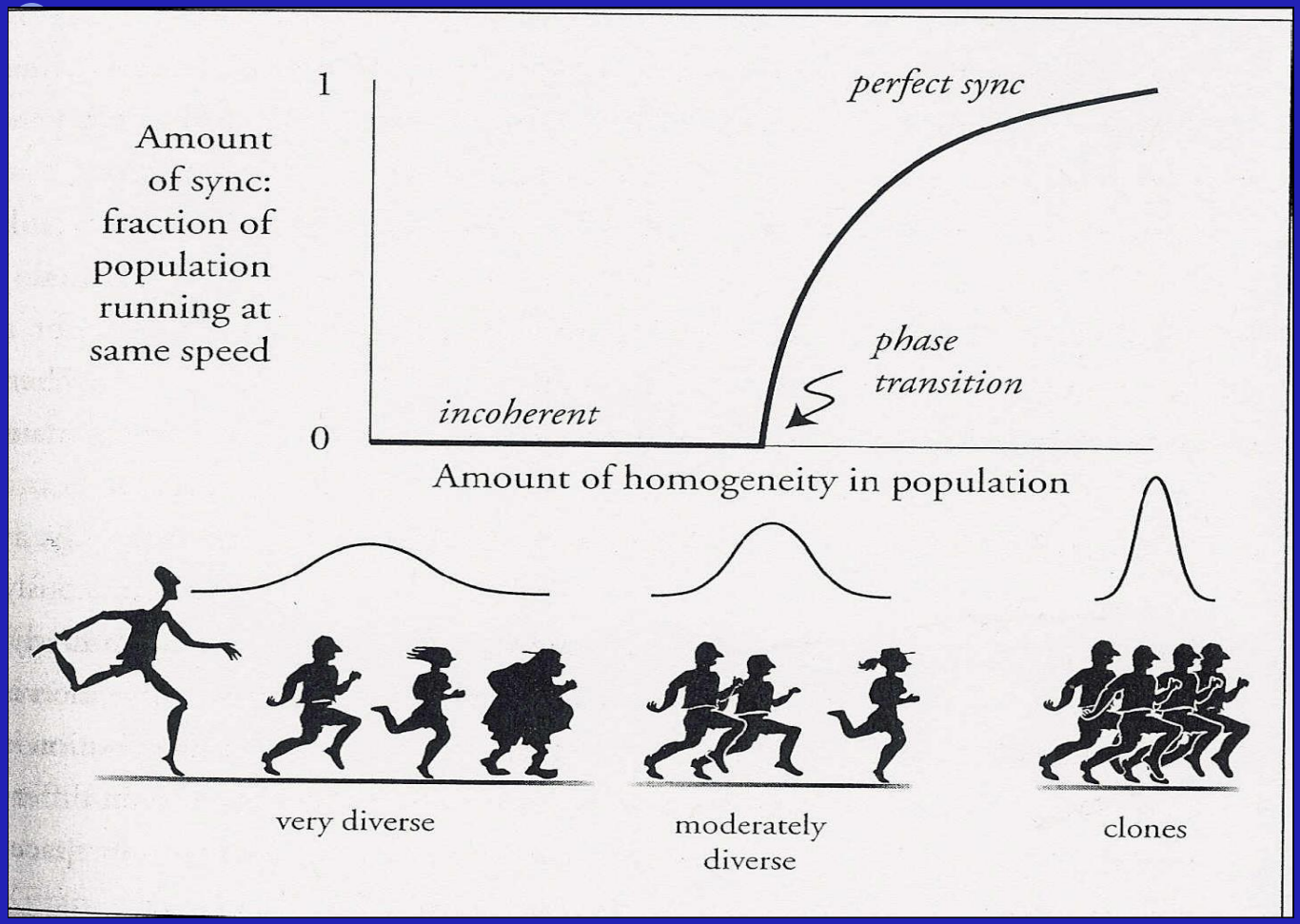

Hurricane and Earthquake Losses 1900-1989
Flood Losses 1986-1992
Cumulative Number of Events / Year
10
1
0.1
0.01
Floods
D = -0.74
Hurricanes
D = -0.98
Eartquakes
D = -0.41
1
10
100
1,000
10,000
Loss Per Event (million 1990 dollars)

sincronizzazione



Richiede forti accoppiamenti non-lineari
Allaccia oscillatori con frequenze differenti

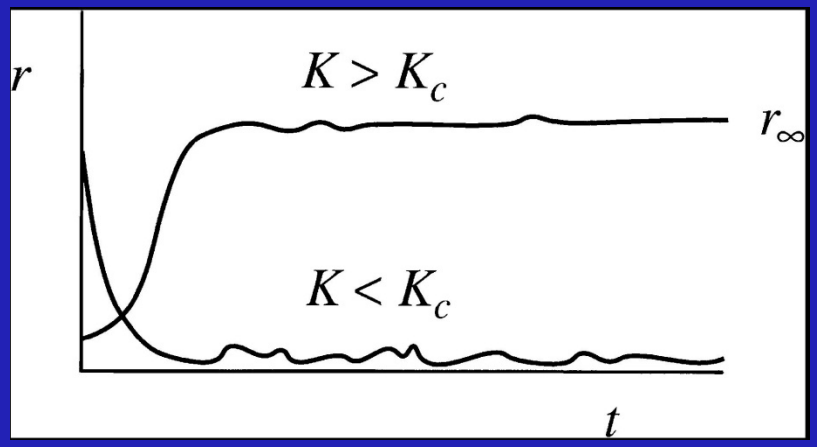
Richiede forti accoppiamenti non-lineari
Allaccia oscillatori con frequenze differenti
Presenta una soglia nel valore di omogeneita'

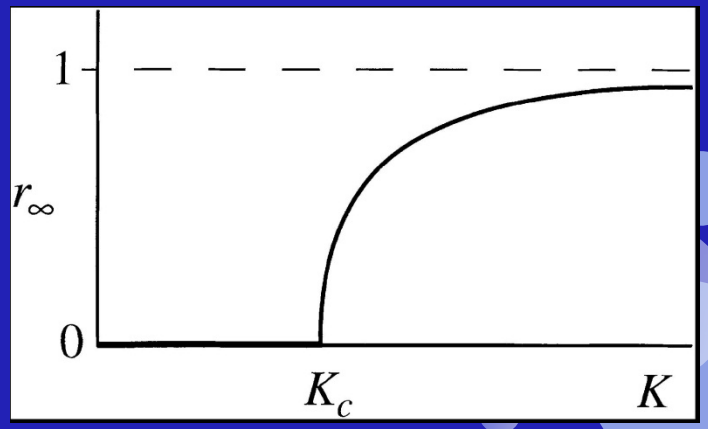


Kuramoto model

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin(\theta_i - \theta_j)$$

$$r(t) := \left|\frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j(t)}\right|$$





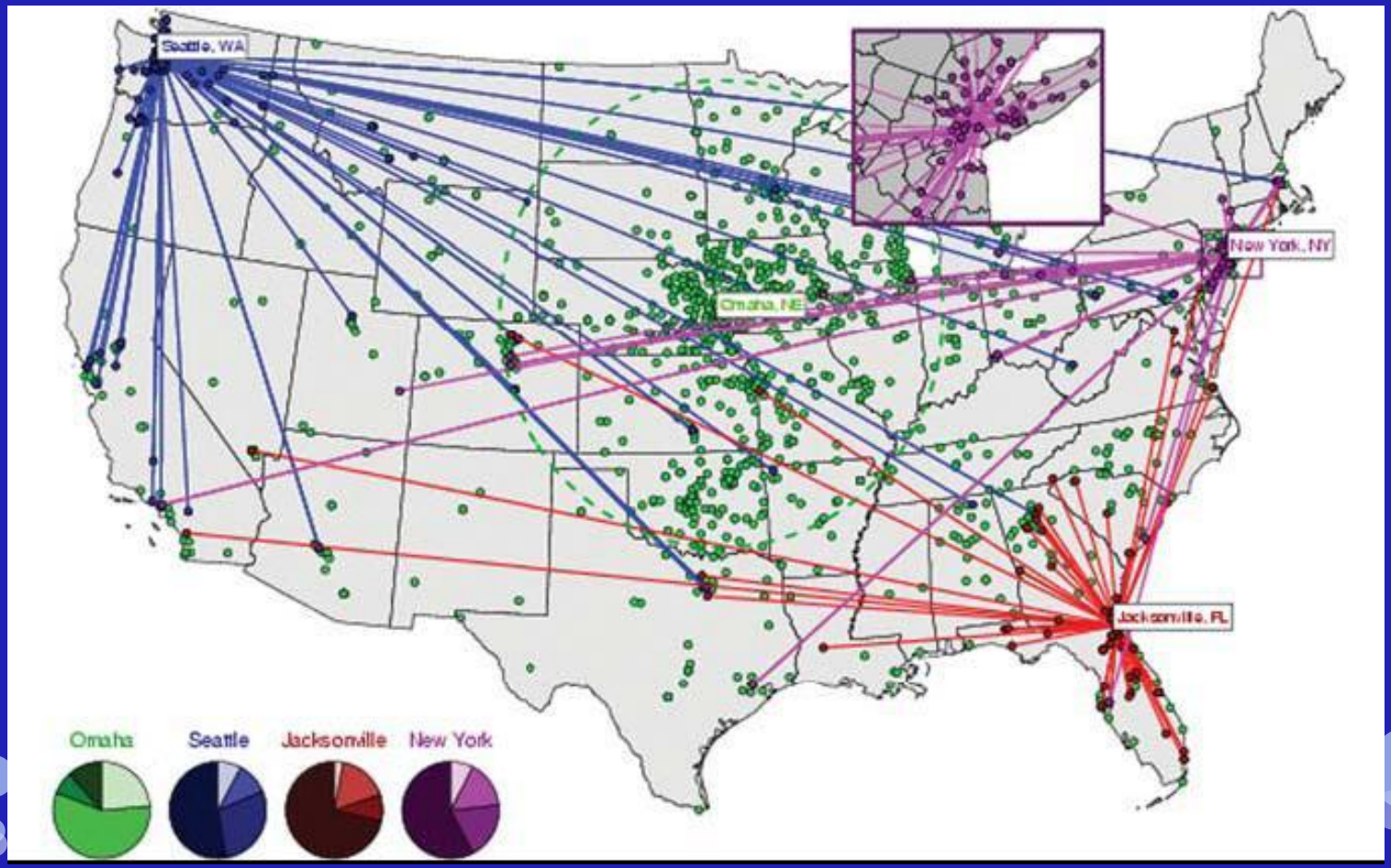
Dinamica e topologia

	Phase-space topology	
dynamics	Lattice/regular	non-lattice/ irregular
Does not modify phase space topology	pattern formation, synchronization, self-organized criticality…	Epidemic spreading, forest fires
Does modify phase space topology	n.a.	Evolution, learning, ..

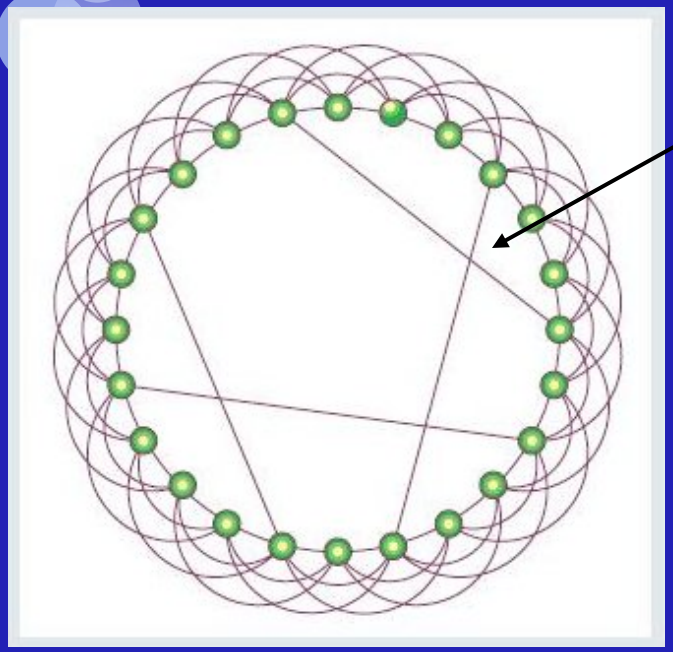
Non-regular lattices

The scaling laws of human travel

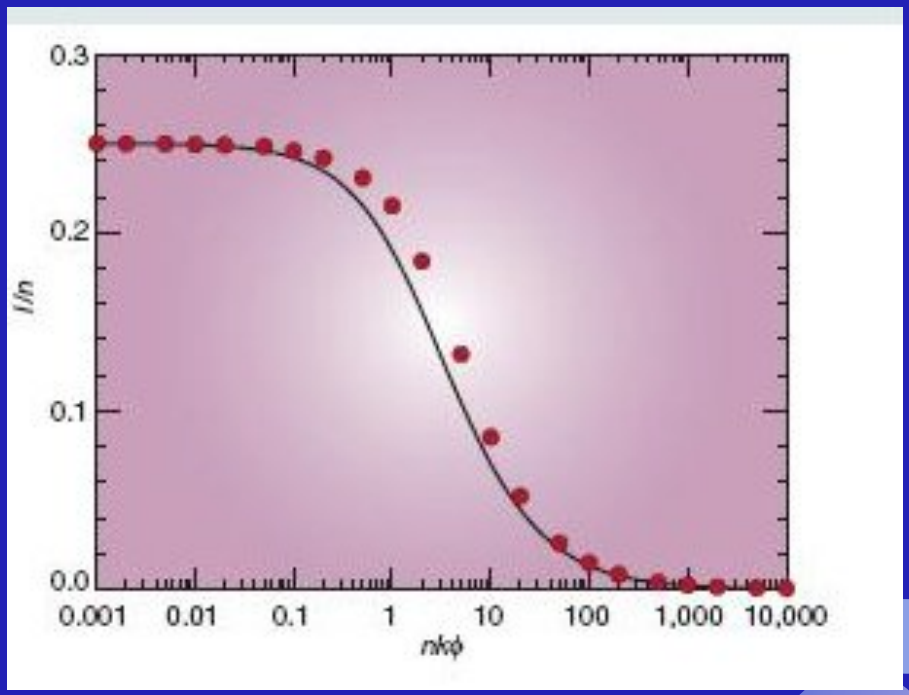
D. Brokman, L. Hufnagel, T. Geisel, Nature 439 (2006)



Piccolo mondo



Si possono stabilire nuovi legami

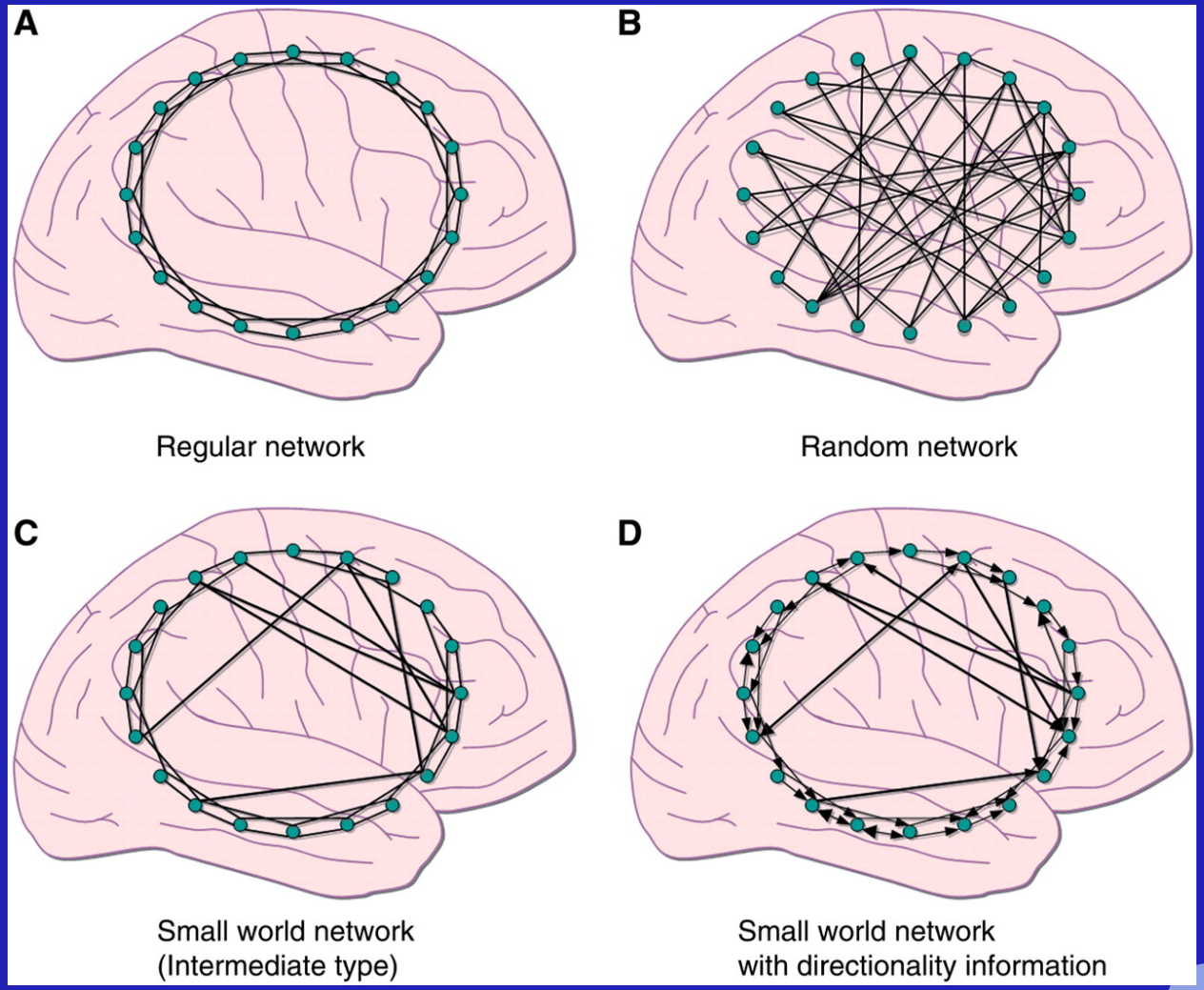


Distanza media

Esempi di piccoli mondi

- Coautori in pubblicazioni scientifiche
- Siti internet
- Giocatori di calcio
- ….

I piccoli mondi presentano SIA allacciamento locale CHE veloce connettivita' globale



Gerloff, C. et al. Brain 2010 133:952-955; doi:10.1093/brain/awq062

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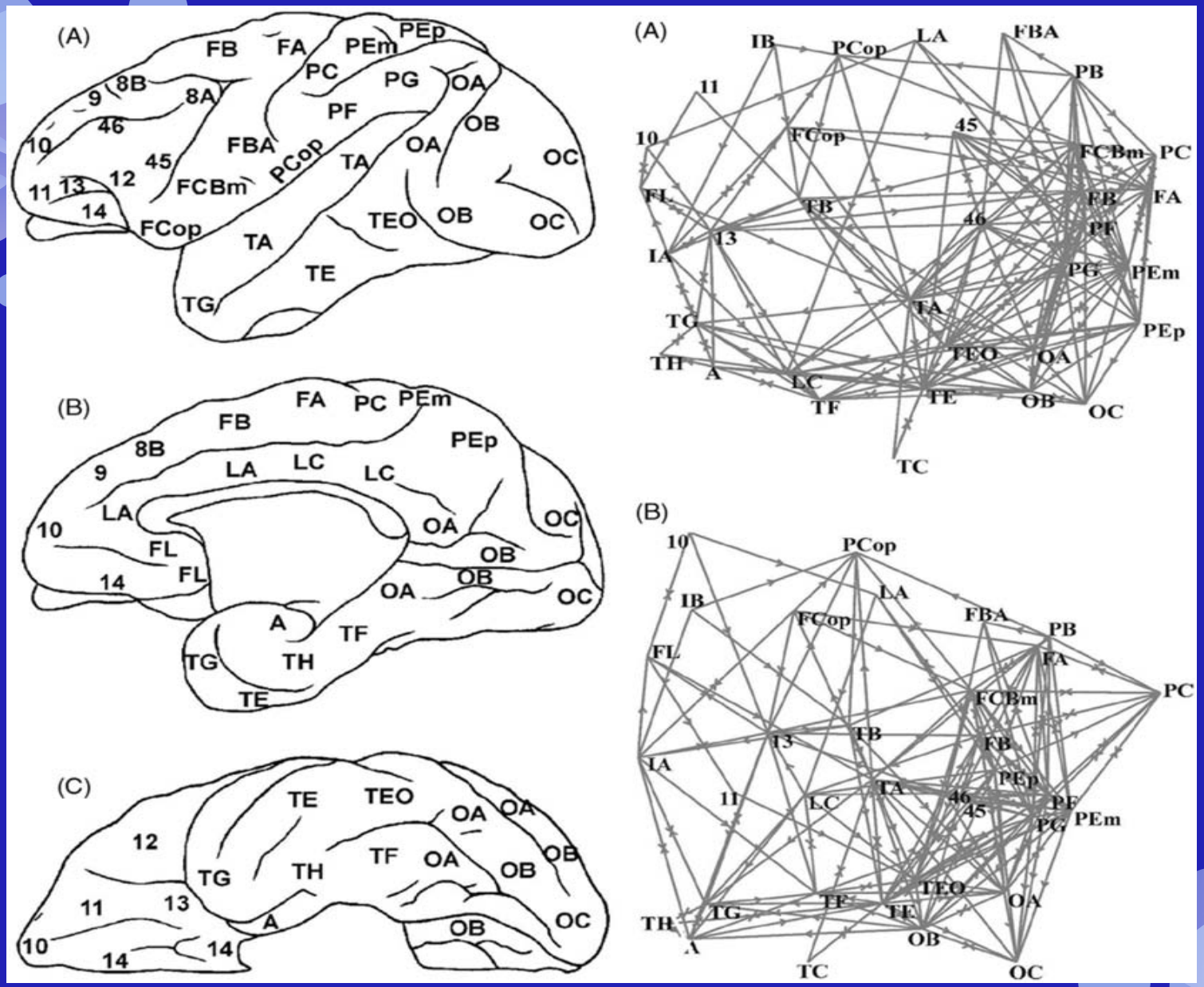
BRAIN A JOURNAL OF NEUROLOGY



Cortical connectivity map

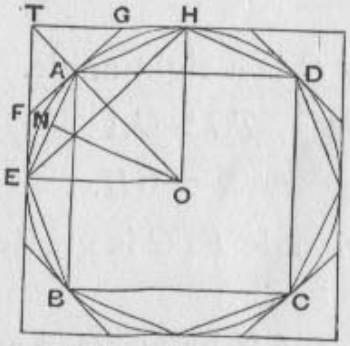
Perimetro ed area del cerchio

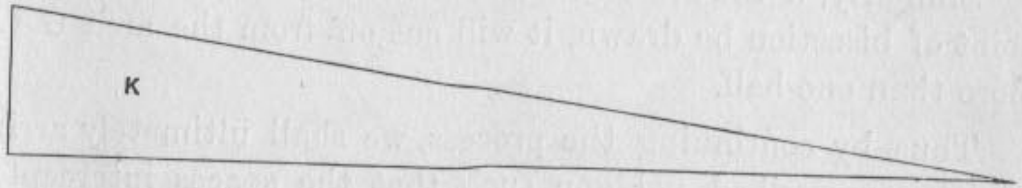
MEASUREMENT OF A CIRCLE.

Proposition 1.

The area of any circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other to the circumference, of the circle.

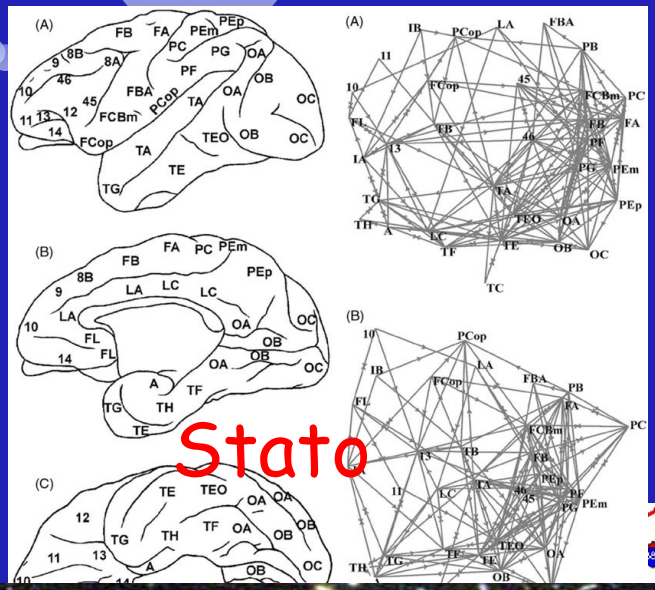
Let $ABCD$ be the given circle, K the triangle described.





Finale

Cervello



Stato

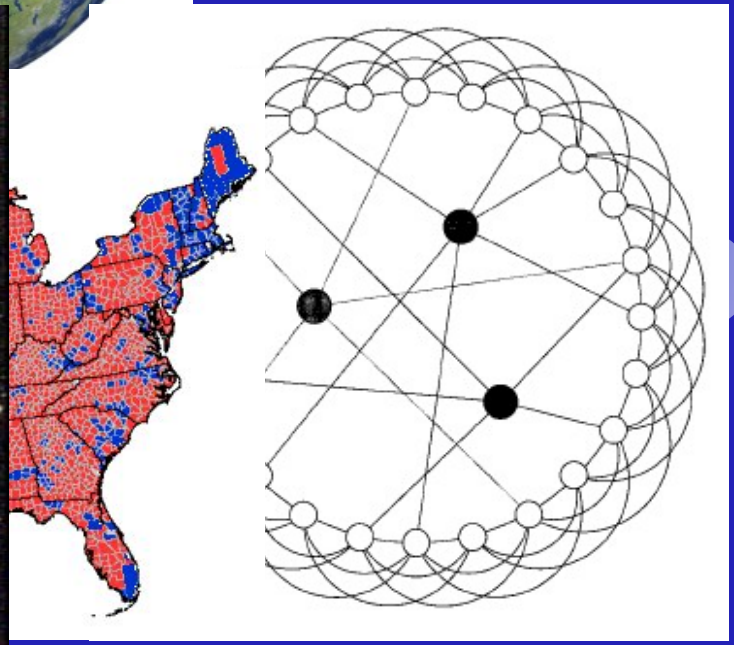
Terra

Strutture gerarchiche

Legami auto-organizzati

Dinamica non-lineare

Nazione Contatti sociali



Universo

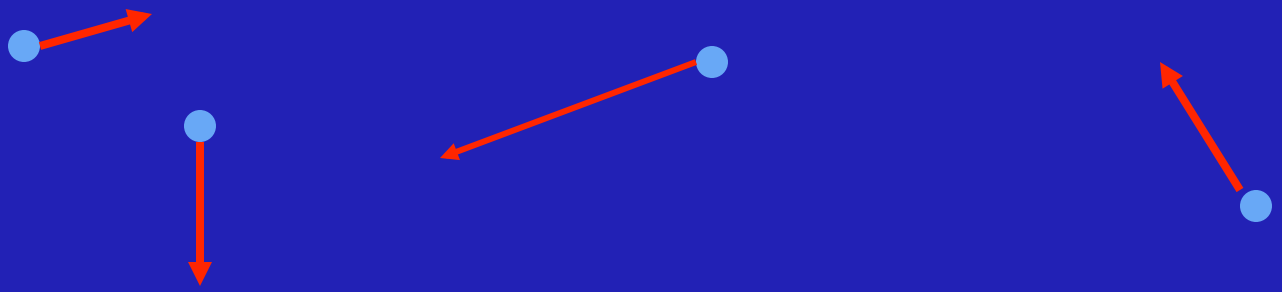
Possiamo descrivere ?

Possiamo misurare ?

Probabilmente …

Il teorema di esistenza ed unicita'

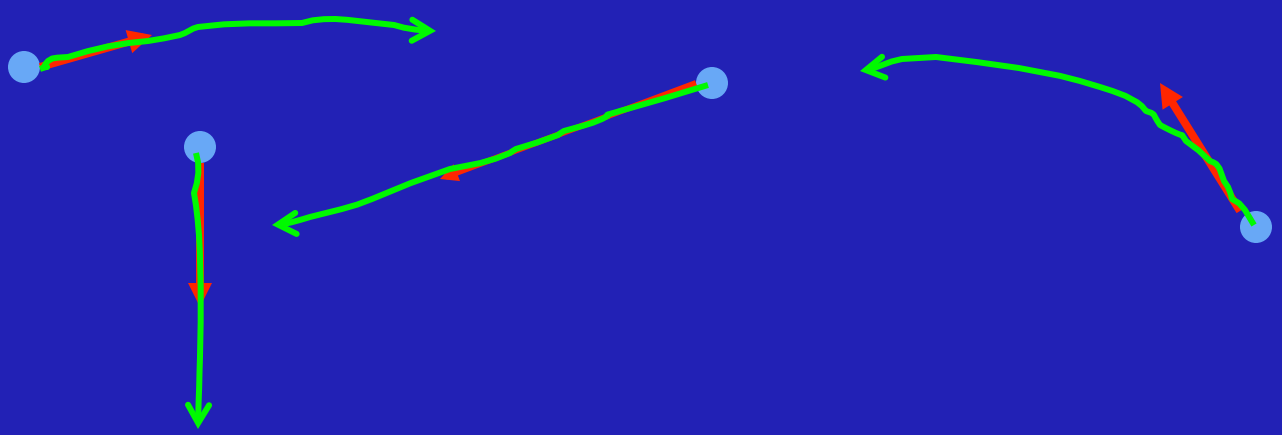
Date N masse $m_1, m_2, \ldots, m_N$ aventi, al tempo zero, posizioni $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$ e velocita' $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_N$



Il teorema di esistenza ed unicita'

risulta univocamente determinata, per tutti gli istanti successivi e precedenti, l'orbita

$$\vec{r}_i(t), \vec{v}_i(t), \quad i = 1,2,\ldots,N$$



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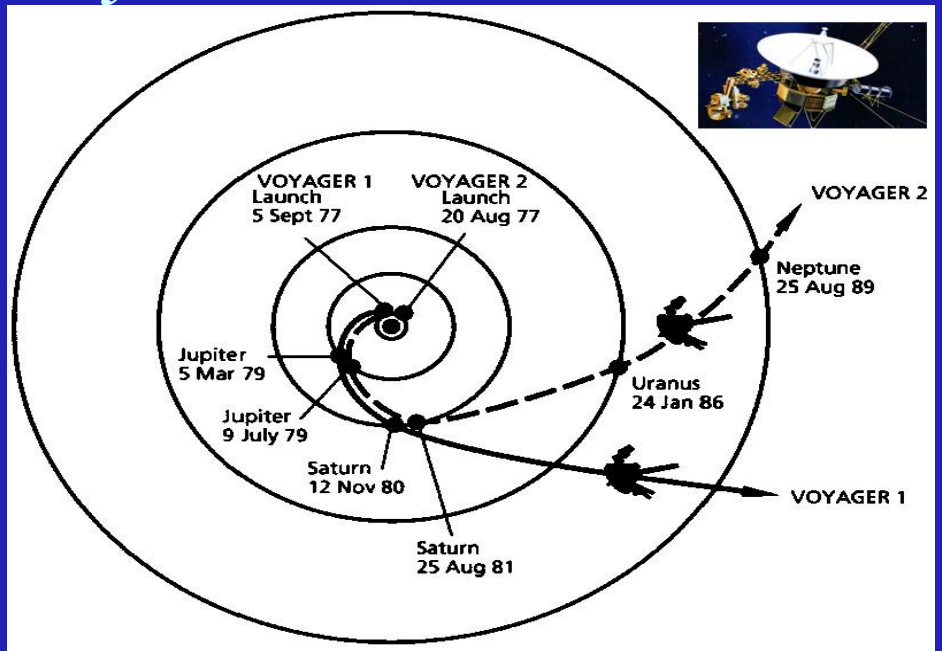
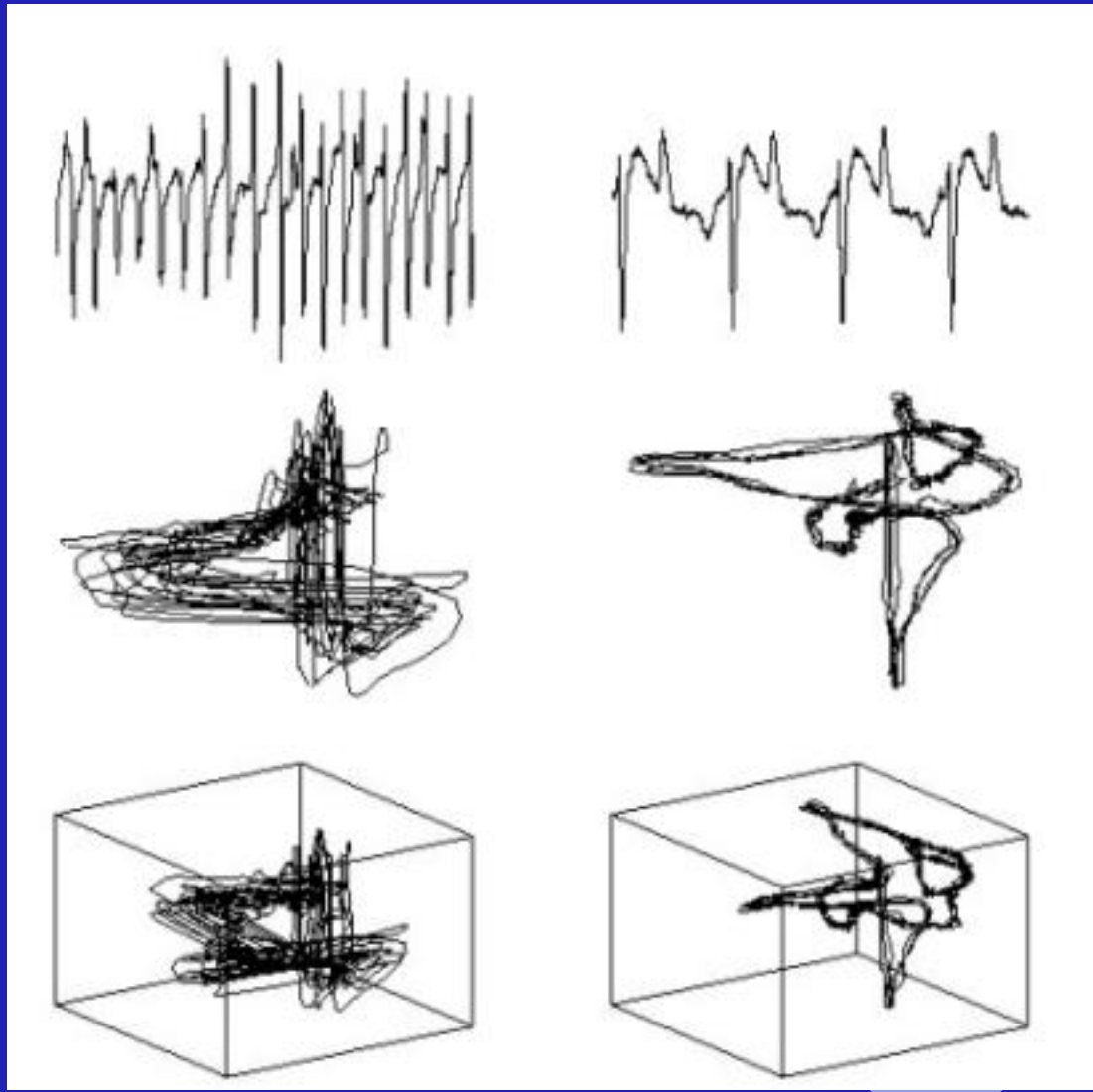


Il caos e' ovunque II

Nel cuore …



Fine…